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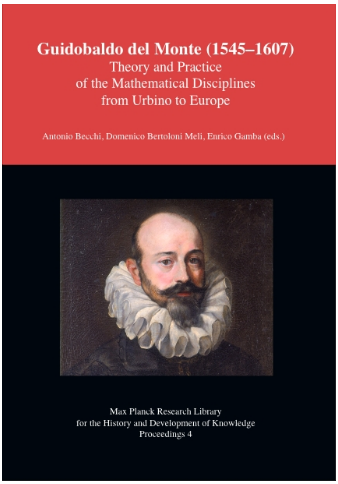

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Chapter 3
Guidobaldo Del Monte's Controversy with Giovan Battista Benedetti on Positional Heaviness

Jürgen Renn and Pietro Daniel Omodeo

3.1 Introduction

Guidobaldo del Monte's handwritten notes on scientific and technical matters *Meditatiunculae de rebus mathematicis* contain, among other things, a criticism of the basic principles of mechanics set down by Giovan Battista Benedetti in *De Mechanicis*. This was printed as the third section of his book *Diversarum speculationum mathematicarum et physicarum liber* (Turin, 1585). Both Benedetti and del Monte are central figures in the history of sixteenth-century science: Benedetti provided an important source for understanding the struggles of early modern engineer-scientists with the ancient heritage of mechanical knowledge from Aristotle, Archimedes and others, whereas del Monte can be regarded as the leading expert on mechanics of the generation before Galileo. Del Monte also authored of one of the most influential texts on mechanics of the early-modern period, the *Mechanicorum liber* (Pesaro, 1577). Del Monte's remarks on Benedetti's book document a disagreement over the conceptual foundations of mechanics and are thus worthy of close consideration for their historical and theoretical meaning.[1]

Let us briefly recapitulate the biographies of the two protagonists of this controversy. Guidobaldo del Monte was born on 11 January 1545 in Pesaro in the territories of the Duke of Urbino. He was taught mathematics by Federico Commandino, who also instilled in him a love of the classics, especially Archimedes. In 1577, del Monte published his first book, the *Mechanicorum liber*, a comprehensive work on mechanics dealing with the five simple machines: the lever, the pulley, the wheel and axle, the wedge and the screw, whose properties were in

[1] This chapter draws on a talk delivered by Jürgen Renn and Peter Damerow entitled "Guidobaldo's marginal notes on Benedetti's *Diversarum speculationum*." It presented for the first time del Monte's marginal notes in his copy of Benedetti's *Diversae speculationes*. These marginal notes have meanwhile been published with a commentary in the Edition Open Access series of the *Max Planck Research Library for the History and Development of Knowledge* under the title *The Equilibrium Controversy*. The loss of our dear colleague Peter Damerow has forced us to shift the focus of this contribution.

turn derived from those of the balance and the lever. In 1581, this work was translated into Italian. In the role of intermediary and patron, del Monte furthered the career of young Galileo, in particular by securing appointments for him, first in Pisa and then in Padua. In later life, del Monte pursued his scientific studies and made scientific instruments at the family castle in Montebaroccio until his death in 1607. In 1592, the year of his move to Padua, Galileo visited del Monte at Montebaroccio and together they performed the experiments on projectile motion that led to the discovery of the law of fall (Renn, Damerow, and Rieger 2001). On that occasion, and possibly even earlier, they must have also discussed foundational issues of mechanics, including Benedetti's approach. The *Meditatiunculae*, bearing witness to del Monte's familiarity with Benedetti's work, is an assembly of writings on a variety of subjects ranging from sundials, astronomy, geometry, perspective, mechanics to optics.[2]

The second protagonist of our controversy, Giovanni Battista Benedetti was born in Venice on 14 August 1530. He was first educated by his father and, according to a brief autobiographical remark of Benedetti, later studied the first four books of Euclid's *Elements* under the guidance of Niccolò Tartaglia, probably between 1546 and 1548. Tartaglia may have also introduced the young Benedetti to the problems of mechanics as he had treated them in his own book, *Quesiti et inventioni diverse* (1546). In 1558, Benedetti joined the court of Ottavio Farnese, the duke of Parma, as an engineer-scientist, and in 1567 moved to Turin to the court of Emanuele Filiberto, the duke of Savoy. He died in Turin on 20 January 1590. Before publishing his major work on mechanics, *Diversarum speculationum mathematicarum et physicarum liber* in 1585, he had written a number of works dealing, among other topics, with geometrical problems, falling bodies and sundials. *Diversae speculationes* first appeared in Turin and was reissued in Venice in 1586 and in 1599. The work comprises the following six treatises: on arithmetical theorems, on perspective, on mechanics, on certain opinions of Aristotle (in particular, the theory of motion), on the fifth book of Euclid and on physical and mathematical problems.[3]

A recent analysis of the Renaissance controversies on equilibrium centers on the edition of the marginal notes that del Monte made in his personal copy of Benedetti's book (Renn and Damerow 2012). Against the background of this study, it is now possible to reassess the historical and theoretical significance of the pertinent remarks made by del Monte in his *Meditatiunculae*. These remarks are very close to the marginal observations he made in his copy of Benedetti's

[2] A first analysis and an overview of del Monte's handwritten work has been carried out by Roberta Tassora in her Ph.D. dissertation (Tassora 2001). This is freely available from the ECHO website at http://echo.mpiwg-berlin.mpg.de/content/mpiwglib/pesaro/#tassora.

[3] The last treatise is dealt with in many letters.

Diversae speculationes, so that the two sets of texts appear to illuminate each other. We will therefore discuss these *marginalia* and the passages of the *Meditatiunculae* on Benedetti at the same time. Moreover, between the folios of the *Meditatiunculae* in question (ff. 145 and 146), dealing with chapters two and three of Benedetti's *De Mechanicis* respectively, one finds an inserted sheet (f. 145*bis*) with a drawing of Galileo's famous comparison of the inclined plane with the bent lever. This insertion does not seem to be cursory since the problem of the bent lever is also relevant for del Monte's analysis of Benedetti's passages. Additionally, del Monte's familiarity with *Diversae speculationes* as well as his criticism of Benedetti's viewpoints sheds new light on a controversial issue of the history of science: the relationship between Benedetti and Galileo. These documents in fact bear indirect evidence of Galileo's acquaintance with the theories of Benedetti. At the same time, they explain his reluctance to mention Benedetti who was regarded critically by Galileo's friend and supporter del Monte.

3.2 The Incipit of Benedetti's *De mechanicis*

Benedetti's book on mechanics begins with a brief introduction that is significant in that it reveals a strong "modernist commitment." The author is convinced that the advancement of science is a future-oriented process in which novelty plays a crucial role. As a scholar of mechanics, he acknowledges to owe much to the work of past generations (*scripserunt multi multa*). Yet, he maintains that nature and practice (*natura ususque*) always bring to light something previously unknown. Accordingly, he promises to provide those interested in mechanical problems (*his qui in hisce mechanicis versantur*) with new insights or, in his words, "things that have never been tried nor explained with sufficient accuracy before" (Benedetti 1585, 141). The importance that he attaches to his treatise *De mechanicis* is evidenced by his hope and expectation that future generations would remember him for his scientific achievements in the field of mechanics. Before dealing with the foundational principles of mechanics, Benedetti stresses the unprecedented originality of his contributions but makes no explicit reference to any forerunners or contemporary scholars. Del Monte, who had published his *Mechanicorum liber* only a few years earlier, would undoubtedly have been offended by this omission. In addition, Benedetti's "progressivist" conception of scientific research potentially contrasted the past-oriented idea of knowledge as a restoration that prevailed during the Renaissance. This aspect could also mark a profound disagreement between his own and del Monte's (and the Commandino school's) purist understanding of science as a restoration of classical sources through accurate philological and mathematical work in the wake of Archimedes.

3.2.1 *Pondus* and *Gravitas*

The first section of Benedetti's *De mechanicis* presents the basic thesis that the weight of a body placed at the extremity of a balance varies in relation to the different inclinations of the beam. This idea goes back to the medieval *scientia de ponderibus* and, in particular, to the work of Jordanus Nemorarius (thirteenth century) who authored a very influential text on weights, the *Liber de ponderibus* (1533). At the beginning of chapter one of his book on mechanics, Benedetti talks of a varying quantity of heaviness, or gravity (*gravitas*), belonging to a weight (*pondus*) or a body placed on a balance beam. The terminological distinction between *pondus*, as a kind of absolute weight or heavy thing, and *gravitas*, as a downward tendency that can act with more or less force on the body (depending on the inclination of the beam), is not rigorous. Benedetti treats the *pondus* at times as the varying quantity to be taken into consideration, as is shown by expressions like "*proportio ponderis in C ad idem pondus in F*" and "*unde fit [...] pondus magis aut minus grave*," in *De mechanicis*, chapter II (Benedetti 1585, 142). Given these semantic fluctuations, we will translate *pondus* as "body" or as "weight" and *gravitas* as "heaviness" or as "weight," depending on the context.

The essentialist meaning Benedetti attaches to the term *pondus* can be traced back to an implicit scholastic background: *pondus* is a "substance" (in the Aristotelian meaning of *hypokeimenon*) while *gravitas* is its "accidental" property, which can be increased or diminished without affecting the essence. In other words, we are confronted with an Aristotelian treatment of quantity (in this case the *gravitas*) as the propriety of an entity (in this case the *pondus*), whose degree of heaviness varies in a qualitative manner.

The profound relation of Benedetti's conception with scholastic Aristotelianism emerges even more clearly when one considers that the concept of *gravitas secundum situm* from which his conception derives was itself shaped by Aristotelian logic. More specifically, the concept of *gravitas secundum situm* can be understood as having been introduced in thirteenth-century mechanics to avoid fallacies that could arise without such a differentiation and specification of the concept of weight.

Aristotle dealt with such fallacies in *On Sophistical Refutations*, V (166b36–167a14). The fallacy relevant to the medieval differentiation of the concept of weight is the *fallacia secundum quid*, referring to erroneous reasoning based on inappropriate generalization. Petrus Hispanus, a contemporary of Jordanus Nemorarius, explained the meaning of this fallacy in his logical treatise *Tractatus sive Summule Logicales* (Hispanus 1972, 157–158). In this book, *secundum quid* means either a "diminution" of a concept through restriction of its definition (*secundum quid et simpliciter*), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*). A fallacy *secundum* *quid* occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (that is, *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument "*est animal pictum [...] ergo est animal*" is fallacious.[4]

The analogy with the concept of *gravitas secundum situm* is evident: according to the doctrine of positional heaviness, weight must be considered in a particular respect, that is, in dependence of its collocation. Just as a general concept has sometimes to be subjected to a restriction of its meaning (*diminutio*) by considering it *secundum quid* before any conclusions can be drawn, so the concept of weight has also to be specified with regard to its collocation before reaching any conclusions, for example, about the equilibrium of a balance.

It should be remarked that according to scholastic logic the determination of heaviness *secundum situm* does not allow conclusions to be made about the absolute weight of a body. The acknowledgment of the relativity of weight, depending on the specification, would eventually undermine Aristotelian natural philosophy on the basis of considerations derived from Aristotelian logic. Benedetti for instance, connected the relativity of heaviness not only to positional heaviness, but also to the medium in which a body is submerged and moves. In the fourth book of *Diversae speculationes*, entitled *Disputationes de quibusdam placitis Aristotelis*, he famously based his treatment of the motion and the fall of bodies through a medium on Archimedean hydrostatics. This theoretical background permitted him (as later Galileo) to relativize heavy and light, depending on the density of the medium. Moreover, he considered the resistance of the medium as a factor to be taken into account in dynamics and thereby reassessed the existence, and even the necessity, of void in nature. Paradoxically, Benedetti (and later Galileo) attached to this Archimedean research agenda a clear anti-Aristotelian significance, although as we have shown, the idea of determining weight *secundum quid* (the *quid* being a factor like position or medium) was directly derived from Aristotelian concepts.

3.2.2 *De mechanicis*, Chapter I: "On the different positions of the beams of a balance"

In chapter I, Benedetti notes that "a body (*pondus*) [...] acquires a larger or smaller weight (*gravitas*) depending on the different ratio of the beam's position" (*pondus [...] maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii*). According to Benedetti, a body has the greatest heaviness when the

[4] We are grateful to Dominik Perler for a helpful discussion of this point, as well as for his suggestions of pertinent literature.

beam at whose extremity it is loaded is in the horizontal position. His idea is based on a simple common-sense intuition: if one considers an equal-arms balance suspended at its center, the weight of a loaded body:

- is borne entirely by the fulcrum when resting vertically upon it,
- is entirely hanging on the fulcrum when suspended vertically below it,
- is not supported in any way by the fulcrum when the beam is in the horizontal position.

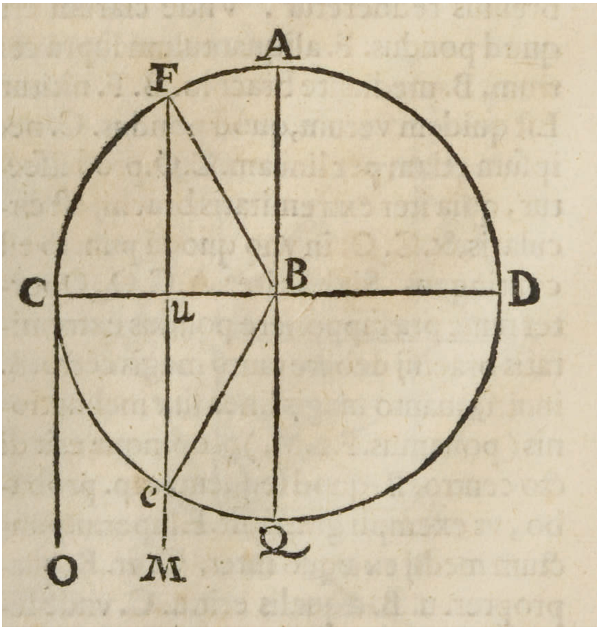

Figure 3.1: Figure in Benedetti's *De mechanicis*, Chapters I and II.

In the first case, the body completely rests or leans on the center (*nititur*), and the center in turn hinders (*impellet*) the downward tendency of the weight. In the second case, the body is suspended vertically (*pendet*) and the center "attracts" it (*attrahet*), in the sense that it hinders its natural tendency to fall down (*inclinatio*). Hence, the body attains its maximum weight in the third case. If the beam of a balance moves upward, departing from the horizontal position, the weight slowly decreases and reaches its minimum at the top when the beam is in the vertical position. If the rotatory motion around the fulcrum continues, now downward, the weight increases again until it reaches its maximum in the horizontal position. It then diminishes until it is suspended entirely below the fulcrum. Benedetti visualizes these variations of weight in dependence on the position (*situs*) thanks to a diagram comparing the lines connecting the weight to the center of the world in different cases, and precisely if the beam:

- is horizontal,
- is raised upward, or
- (which is equivalent to the second case) is moved downward with the same angle as in the second case.

The parallel lines, called *lineae inclinationis* or *lineae itineris*, indicate the direction in which a body would fall if it were free. The closer these lines are to the center of the beam, Benedetti says, the "less heavy" the body becomes (Figure 3.1).

In his copy of Benedetti's book, del Monte wrote a brief annotation in the margin of chapter one: "this first chapter is derived entirely from our treatise on the balance in the *Mechanicorum liber*."[5] Clearly, he vindicated the relevance of his treatise for Benedetti's speculations, in spite of the latter's claims of originality. It should be remarked, however, that del Monte's treatment of the balance, based on the concept of center of gravity, was significantly different from Benedetti's, which was based on an original reworking of *positional heaviness*. For del Monte though, he merely reassessed a concept received from authors such as Jordanus Nemorarius, Tartaglia and Cardano, all of whom he personally opposed. In his book on mechanics, del Monte had in fact criticized the concept of *positional heaviness*. Downplaying Benedetti's theory as a repetition of his predecessor's theories, he could therefore claim that his own treatment already included a résumé (as well as a criticism) of Benedetti's approach.

3.2.3 *De mechanicis*, Chapter II: On the proportion of weights at the extremities of a balance beam in a position other than the horizontal

In chapter II, Benedetti deals with the proportions of a weight placed at the extremity of a balance beam if its position is not horizontal (*De proportione ponderis extremitatis brachii librae in diverso situ ab orizontalis*). The thesis to be demonstrated is the following: "The proportion between [the weight of] a body (*pondus*) at C and [the weight of] the same body (*pondus*) at F corresponds to that between the whole beam BC and its part BU which is [set on the beam BC and is] delimitated by the fulcrum and the [intersection between the beam and the]

[5] "Hoc primum caput to[tum] desumptum est a n[ostro] *Mechanicorum libri* tractatu de lib[ra]."

inclination line *FUM* that connects the weight at *F* to the center of the world" (Benedetti 1585, 142. Figure 3.2).

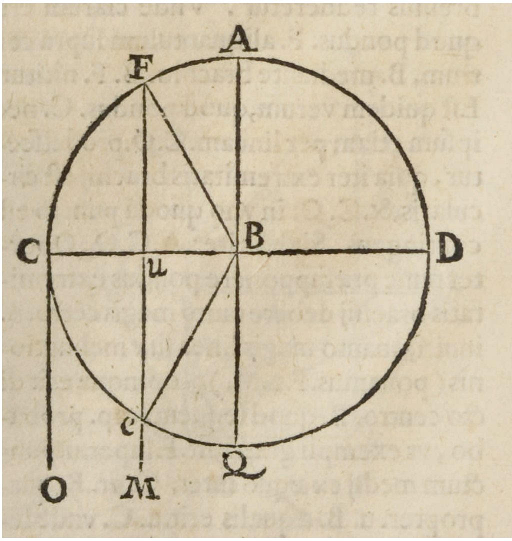

Figure 3.2: Benedetti discusses in *De mechanicis*, chapter II, the same figure as in *De mechanicis*, chapter I.

For the sake of simplicity, we represent these relations symbolically, in modern terms:

$$C : F = BC : BU$$

where C is the weight in the horizontal position and F in the inclined position; BC is the beam and BU the part of the beam BC between the center B and the perpendicular line drawn from F.

Benedetti's demonstration is as follows. He imagines placing a weight D on the other extremity of the balance that has the same proportion to C as F, that is, the following proportion expressed in modern terms:

$$D : C = BU : BC.$$

In accordance with Archimedes's *De ponderibus* I.6, the balance will be stable if the weight C is loaded at U since weights and distances from the fulcrum are proportional by supposition.

The next step is to show that $F : C = BF : BU$ (where BF is the beam, hence $BF = BC$). In order to demonstrate this, Benedetti resorts to the mental model (*imaginemur*) of a string hanging vertically from F to which a weight equal to C is suspended. He claims that it is visually evident that the weight has the same effect at F as at U. The same is valid for the case in which the weight is suspended from U and intersects the circumference described by the rotation of the beam at a point E. In both cases, the balance would remain horizontal since the weight C at F, U or E would balance the weight at D. Benedetti further argues that the balance under consideration can be treated like a bent lever with a horizontal and an inclined arm (*FBD* or *EBD*): "*si brachium* BE *consolidatum fuisset [...]*" (If the beam *BE* was made solid [...]).

The author concluded that his reasoning has satisfactorily demonstrated his thesis: "A body (*pondus*) is more or less heavy (*grave*) the more or less it hangs from (*pendet*) or rests on (*nititur*) the fulcrum" (Benedetti 1585, 142). And he deems this resting on or hanging from the fulcrum to be the most direct cause (*haec est causa proxima, et per se*) of the positional changing of a weight.

As an additional commentary, Benedetti remarks that in his diagram he supposes the inclination line *CO* to be perpendicular to *CB* and parallel to *BQ*, whereas *CO* and *BQ* in fact converge at the center of the sphere of the elements (*centrum regionis elementaris*), that is, the Earth. But for the sake of his present argumentation, this angle is negligible and one may remain with the assumed perpendicularity and parallelism. Benedetti thus developed a method to quantify positional heaviness that corresponds to the modern concept of "torque."

3.3 Del Monte's Rebuttal of the Negligibility of the World's Center

As will be shown in the following, only in his initial treatment of the inclined balance did Benedetti neglect to consider the convergence of the inclination lines to the center of the elements, that is, in chapter one of *De mechanicis*. This omission gave rise to criticism in the *Meditatiunculae* by del Monte, who, to dispel Benedetti's reasoning in *De mechanicis*, chapter I, did not accept his premises. Rather, he assessed Benedetti's arguments from his perspective, relying on the idea of center of gravity as developed in his own book on mechanics.

A marginal note by del Monte documents his disagreement with Benedetti's conclusion: "Thus, in this manner, a weight (*pondus*) more or less hangs from or rests on the center; this is the next cause and the [cause] in itself [of the variation in heaviness]."[6] As a marginal note (Figure 3.3), he wrote:

> because that [that is, the greater or smaller extent to which a weight rests at the center] is neither the next [cause] nor the [cause] in itself. For the weight at *F* of the arm *BF* is not equally heavy as the weight *U* of the arm *BU*; nor is the weight at *E* of the arm *BE* equally heavy as the weight at *U* of the arm *BU*. Thus, this entire demonstration is false.[7]

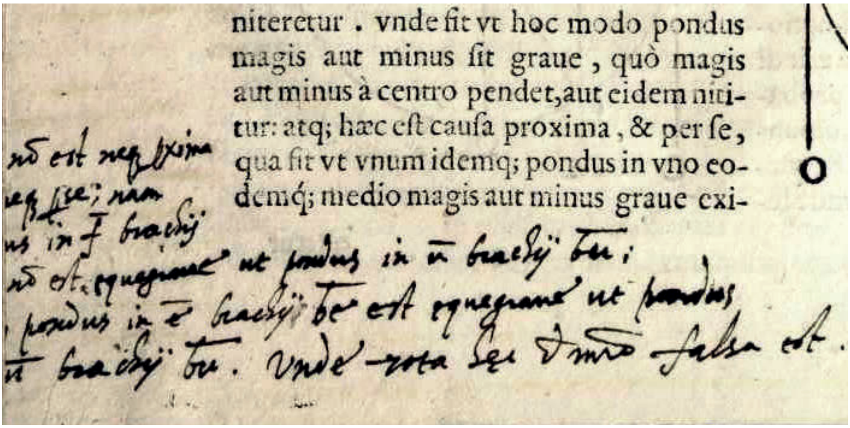
niteretur . vnde fit vt hoc modo pondus
magis aut minus sit graue , quò magis
aut minus à centro pendet,aut eidem niti-
tur: atq; hæc est causa proxima , & per se,
qua fit vt vnum idemq; pondus in vno eo-
demq; medio magis aut minus graue exi-

Figure 3.3: Del Monte's marginal note to *De mechanicis*, chapter II.

This means that del Monte did not accept the claim that a weight is equally heavy in different positions on the beam of the balance, provided the projections of the beam along the horizontal are the same length or rather, as Benedetti writes, the distances between the projections of the beam on the horizontal and the center have the same lengths.

To find del Monte's counter-arguments, we shall look to the *Meditatiunculae*, f. 145, *Contra Cap. 2 Jo. de Benedicti de Mechanicis*. As mentioned, he basically rejected Benedetti's perspective by objecting that he did not take into

[6] "[…] unde fit ut hoc modo pondus magis aut minus a centro pendet aut eidem nititur: atque haec est cause proxima, et per se [...]."

[7] "non est neque proxima neque per se; nam [pond]us in *F* brachii [*BF*] non est equegrave ut pondus in *U* brachii *BU*; [nec] pondus in *E* brachii *BE* est equegrave ut pondus [in] *U* brachii *BU*. Unde tota haec demonstratio falsa est" (Renn and Damerow 2012, 207).

due account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other, as Benedetti assumed in this part of his treatise.

In his diagram (Figure 3.4), del Monte compared the line *LUS* parallel to the line *AQ*, connecting the fulcrum *B* of the balance with the center of the world *M*, with the line *FM* connecting the upper weight *F* and the lower weight *E* with the center of the world *M*. *S* is the point at which the line *LUS* meets the circle that the beam makes around the fulcrum, which is above the position of the lower weight *E*.

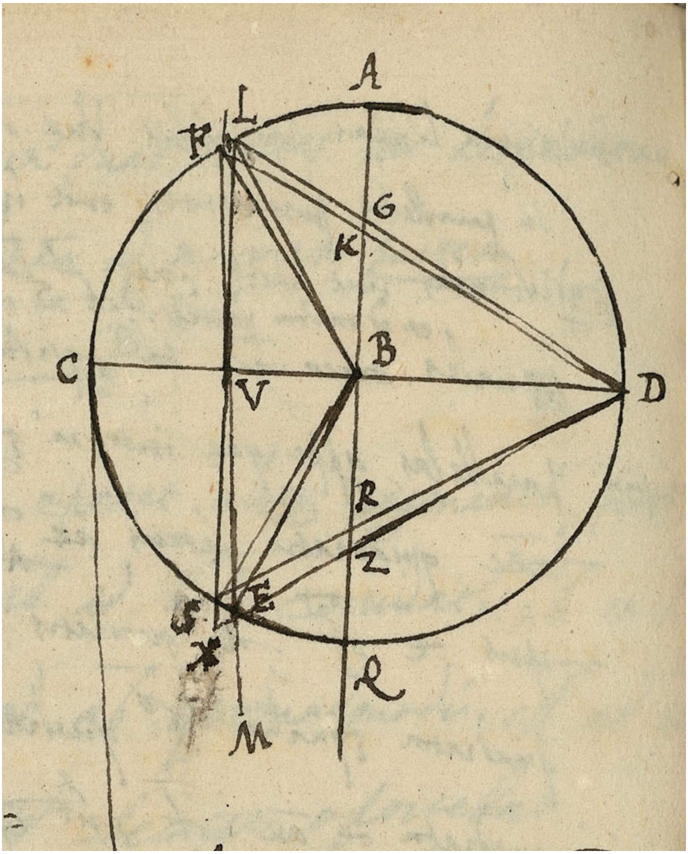

Figure 3.4: Del Monte's reconsideration of Benedetti's analysis of the bent lever.

He next considered a bent lever made of the oblique arm *BS*, rigidly connected to the straight arm *BS*, assuming that *BU* is half *BD*. If a weight is now placed at *S* that is double the weight at *D*, the bent lever will be in equilibrium, as del Monte showed with reference to his book, because the center of gravity of the weights at *S* and at *D* will be at the point *R*, which will be in its lowest place on the vertical line *BQ*. He therefore concluded that it is the weight at *S*, but not the lower weight *E*, that will be equally heavy as the weight at *U*.

He proceeded to demonstrate this in greater detail by considering the proportions in which the line connecting the two weights is cut by the perpendicular *BQ* for the two cases, that is, the weight placed at *S* and the weight placed at *E*. Del Monte concluded that the same weight is heavier at *S* than at *E*. He then turned to a closer consideration of the upper weight *F*. Again he constructed a bent lever *LBD* in equilibrium in order to compare it with the bent lever formed with the upper weight *F*. Again he showed that the weight is heavier at *L* than at *F*.

Del Monte concluded by summarizing that the entire fallacy is due to Benedetti assuming that the weight at *F* would gravitate in the same way as at *U*, which would only be the case, according to del Monte, if it were to hang freely.

3.4 Benedetti's Generalization: From Weights to Forces

Chapter three of Benedetti's *De Mechanicis* contains a generalization of the results of chapter two or, rather, presents a general rule concerning the action of forces (*virtutes*) on the beams of a balance, also in the case that they do not act vertically downward but also with an acute or obtuse angle (Figure 3.5). Benedetti resumes the result of the previous chapter as follows: the length of the line perpendicularly connecting the center to the line of inclination (the line *BU* in the diagram) allows the quantity of the positional force (*quantitas virtutis [...] in [...] situ*) of a weight (*F* in the diagram) to be established. Thus, Benedetti calls the positional weight a force and this is the presupposition to generalize from *gravitas* the action of what he calls *virtutes moventes*, or "moving forces." The thesis of this chapter is summarized in its title: "That the quantity of any given weight (*pondus*) or moving force in relation to another quantity can be determined thanks to the perpendicular projections connecting the center of the balance to the line of inclination."

Benedetti draws two diagrams showing a balance at whose extremities two weights or forces act in different directions. At the left extremity *B*, a weight *E* has a downward tendency while, at the right extremity, a weight *C* acts making an acute or an obtuse angle. According to Benedetti, the length of the perpendicular projection drawn from the center to the inclination line, *OT*, permits the determination of the distance *OI* on the beam at which the same force acting vertically downward produces the same effect. Given this equation, Benedetti can determine how much the force acting in a non-perpendicular direction has to be augmented in order to balance an equal weight acting perpendicularly on the opposite beam. This measure is given according to the following proportion (expressed in modern terms):

$$E : C = BO : OI$$

where E is the weight acting vertically on the extremity B; C is the *virtus movens* acting on the opposite extremity A with an angle; BO is the left beam and OI the part of the right beam OA determined as explained above.

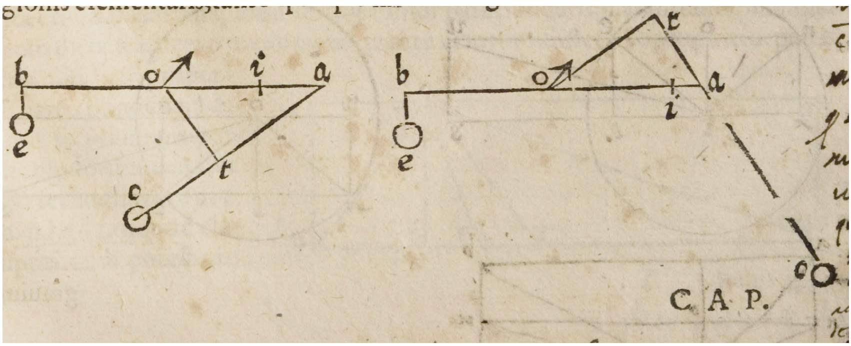

Figure 3.5: Benedetti's representation of forces acting on a balance in arbitrary directions.

In his argumentation, Benedetti thus equates a balance (*BOI*) with a bent lever (*BOT*). Accepting this equation, he concluded that, according to commonly shared knowledge (*communi quadam scientia*), the weights or forces that are required to obtain a perfect balance can easily be calculated.

The chapter ends with a cosmological corollary: "The closer the center *O* of the balance is to the center of the elementary sphere, the less heavy (*minus grave*) it becomes." In fact, the angles between the beam and the inclination lines become progressively smaller.

3.5 Del Monte's Misunderstanding

In his notes on folio 146 of the *Meditatiunculae*, del Monte grappled with Benedetti's instructions of how to determine positional heaviness in the case of forces acting in an arbitrary direction. These he refuted at length under the erroneous assumption that Benedetti had claimed forces can be indiscriminately replaced by weights. Like Benedetti, del Monte considered a bent lever *BOAC* with fulcrum *O*, weights *E* and *C*, a straight arm *BO* and a bent arm *OAC* to discuss the two cases of an acute and an obtuse angle *BAC* (Figure 3.6).

He first recapitulated Benedetti's procedure, assuming that a vertical line *OT* drawn from the fulcrum to the line *AC* represented the oblique arm of the bent lever. He stated that when the weight *C* is placed at the end of the horizontal line *OI*, whose length is the same as that of the perpendicular *OT*, according to Benedetti it will be in equilibrium with the weight *E* if the weight *C* is to the weight *E* as is *BO* to *OT* or *OI*. Del Monte then summarized Benedetti's claim that when a force represented by the weight *C* acts along the line *TC*, the bent lever formed by the straight arm *BO* and the oblique arm *OTC* will also be in equilibrium, which he doubted.

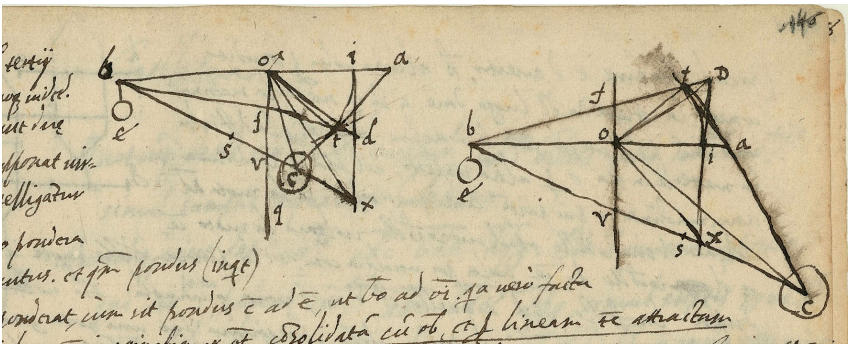

Figure 3.6: Del Monte's critical reworking of Benedetti's representation of forces acting on a balance in arbitrary directions.

Del Monte reformulated this claim by stating that the same weight *C* will be in equilibrium with the weight *E*, whether it is placed on the straight balance *BOI* or on the broken bent lever *BOTC*. He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and as a result arrived at absurd conclusions.

Del Monte then showed that the same weight will be heavier on the horizontal at point *I* than along the bent lever at *T*, demonstrating that the bent lever *TOB* will not be in equilibrium if the straight lever *BOI* is in equilibrium. To show this, del Monte again proceeded by finding the center of gravity of the weights *E* and *C* placed at *T*. More precisely, del Monte determined a position for the weight *C* where the bent lever is in equilibrium, a position, however, that is distinct from *T*. Thus it follows that *T* cannot be the position of equilibrium. For this purpose, he extended the line *BT* to *D*, just beneath *I*, so that it is immediately evident that if the weight *C* is placed at *D*, the center of gravity of the two weights will be just beneath the fulcrum.

Using the same pattern, he continued by showing that the bent lever *BOC* cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum. Finally, he applied this argument to the broken bent lever *BOTC*. Del Monte next addressed the case in which the bent lever is characterized by an obtuse angle *BAC*, showing that the weight at *T* is lighter than the weight at *I*. In his concluding remarks, however, he began to waver. Once again, he stated that Benedetti is completely mistaken when applying his procedure to weights. But he did admit that this may be true when dealing with a force.

As an afterthought, del Monte once again criticized Benedetti's appeal to common sense: he did not feel this to be worthy of an expert mathematician. And as a second afterthought, he constructed an extreme case in which it is immediately clear that the broken bent lever cannot be in equilibrium if weights are attached to it rather than forces.

The following considerations enable del 's marginal annotations to Benedetti's *De Mechanicis*, chapter III, to be understood. These are not perfectly legible, but nonetheless their meaning becomes clear in light of the *Meditatiunculae*:

> If we understand that a weight is at *C*, as we can assume from his own words, then *CT* must also be understood as being solid [and connected with] the solid lines *TO* […] If we hence understand that *C* is a weight and not moving, [the proposition] is false. If it is understood that *C* moves as […] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing […] therefore as is evident in chapter 7. All demonstrations of the author are founded on these two chapters inasmuch as they are the first fundaments of mechanics; once their falsity is recognized, everything is rejected.[8]

[8]"si intelligamus p[ondus] in *C*, ut supponi p[otest] ex verbis ipsius, intelligendum est *C*[*T*] quoque consolidatam consolidatis *TO* […]. Unde si intelligamus *C* pondus et non movens, falsa est i[ta]que si intelligatur *C* movens ut homi[…] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i...] ipse [vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil […] ut patet in 7 cap. In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua

3.6 On Tartaglia: Diverging Approaches

Del Monte's and Benedetti's criticisms of Tartaglia's conception of positional heaviness help us to understand where these two scholars converge and diverge on the issue of the equilibrium (or lack of equilibrium) of a balance deflected from its horizontal position, and also the reasons for the presumed equilibrium or tendency to restore it. Moreover, their arguments reveal a different attitude toward the medieval tradition of the *scientia de ponderibus* and the *gravitas secundum situm*.

3.6.1 The Tradition of Jordanus, Tartaglia and Cardano

The concept of *gravitas secundum situm*, or positional heaviness, was extensively employed in Jordanus Nemorarius's *Liber de ponderibus*. Del Monte owned and annotated a sixteenth-century Nuremberg edition of the book, commented and illustrated by Petrus Apianus. Del Monte's handwritten annotations document his general disagreement with the approach of this scholastic forerunner, who did not know the Archimedean concept of center of gravity and tried therefore to develop a deductive science of weights relying solely on the Aristotelian theory of motion.[9] We have already hinted at the Aristotelian conceptuality underlying the concept of *gravitas secundum situm*. In his book, Jordanus stated that a deflected balance would return to the horizontal position (his second proposition) (Nemorarius 1565, B2 *r*). According to Jordanus, the upper weight acquires more positional heaviness than the lower one due to the fact that its descent is less oblique. In fact, he postulated that positional heaviness depends on the obliqueness of descent of a weight (his fourth postulate) and that "a more oblique descent partakes less of the straight [descent] for the same quantity [of path]" (fifth postulate) (Nemorarius 1533, A4 *r*). The determination and possibly the quantification of obliqueness was therefore essential to establish the behavior of a deflected balance.

In the sixteenth century, Tartaglia in *Quesiti, et inventioni diverse* (1546), and Cardano in the first book of *De subtilitate* (first edition, 1550) and in *Opus novum de proportionibus* (1570), expounded some different methods for determining descent and reinforced Jordanus's second proposition that the deflected balance returns to the horizontal position. A brief account of three ways of determining positional heaviness is given in the following. The first two are derived from Tartaglia and the last from Cardano.

mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur]" (Renn and Damerow 2012, 213).

[9] We are grateful to Martin Frank who discovered these annotations and shared them with us.

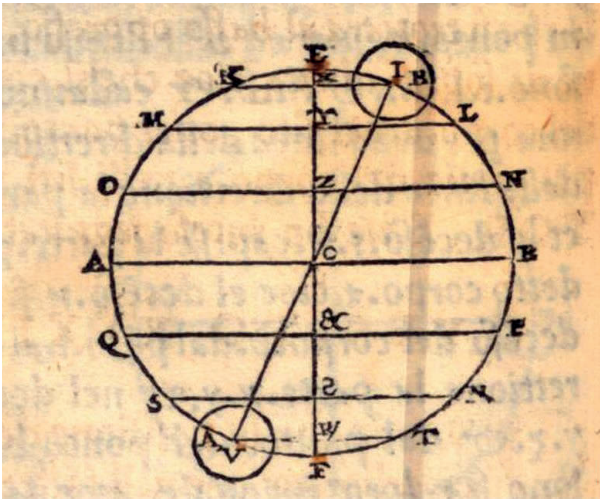

Figure 3.7: According to Tartaglia, the body at *I* is positionally heavier than the body at *V* since the projection of the arc *IL* on the vertical *XY* is greater than the projection of *VF*, *WF*.

DESCENT: A first method of dealing with positional heaviness consisted in comparing the lengths of the projections of the equal arcs described by the motion of opposite balance beams—one ascending and one descending—on the vertical line of descent to the center of the world.

As Tartaglia's diagram shows (Figure 3.7), the vertical component of decent of the upper weight is always larger than that of the lower. Thus, the former acquires more heaviness (*secundum situm*) than the latter and the balance returns to the horizontal position.

ANGLE OF CONTACT: Tartaglia's second method of determining positional heaviness consists in comparing the angles between the circular path of the beams and the perpendicular lines connecting the weights to the center of the elements. These angles "of contact" are also called "curvilinear angles" or "mixed angles" since they result from the intersection of a straight line downward and a curved line, that of the circle circumscribing the balance (Figure 3.8).

By comparing the angles of contact of the two weights, Tartaglia could establish that the higher angle is always smaller than the lower, therefore the higher weight has a straighter descent and is positionally heavier. The inclined balance would therefore return to the horizontal position. It should be noted that Tartaglia perceived the comparison of curvilinear angles as problematic. He considered the ratio of two such angles to be less than any ratio between determined quantities. As a consequence, no weight placed on the positionally lighter side of the deflected balance could compensate for the other weight and keep the balance inclined. On the contrary, any additional weight—no matter how small—would have p roduced an opposite displacement of the balance beam toward the vertical.

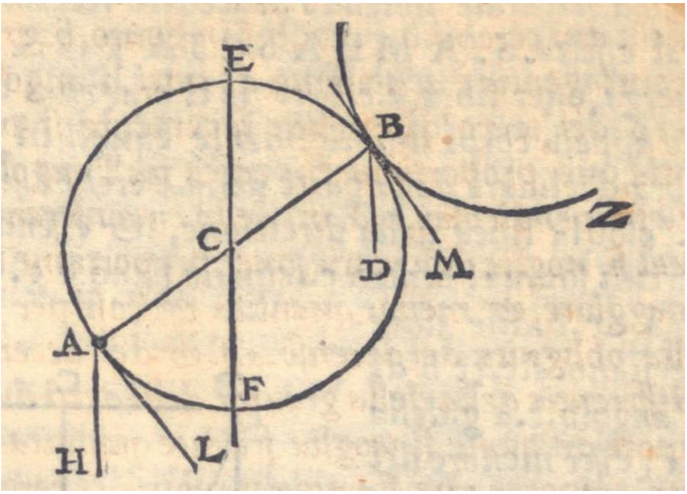

Figure 3.8: According to Tartaglia, the body at *B* is positionally heavier than the body at *A* since the *angle of contact* between *BD* and the arc *BF* is smaller than the angle between *AH* and the arc *AF*.

THE ANGLE BETWEEN THE SUPPORT AND THE BEAMS: We have considered two ways of determining positional heaviness on the basis of Tartaglia's *Quesiti*. Assuming that positional heaviness depends on the obliquity and straightness of descent, positional heaviness can be determined either from the projections of the descents on the vertical, or the curvilinear angles that are produced by the intersection of the descent arcs and the lines connecting the weights to the center of gravity. Cardano considered three criteria for establishing positional heaviness which he mistakenly regarded as equivalent: first, the distance of the beam from the vertical; second, its distance from the horizontal; and third, an angle that he called *meta*. This was the angle between the support of the balance and the beam. Commenting on the diagram that is here reproduced (Figure 3.9), he explained:

> Aristotle says that this happens when the support is above the balance, because the angle *QBF* of the *meta* is larger than the angle *QBR*. And similarly, when the support is *QB*, the *meta* will be *AB*, and thus the *RBA* will be larger than the angle *FBA*, but the larger angle will render the weight heavier. […] The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.[10]

Given these premises, Cardano contended that a weight will reach its maximum positional heaviness in the horizontal position. He therefore shared Nemorarius's and Tartaglia's opinion about the return of an inclined balance to the horizontal position.

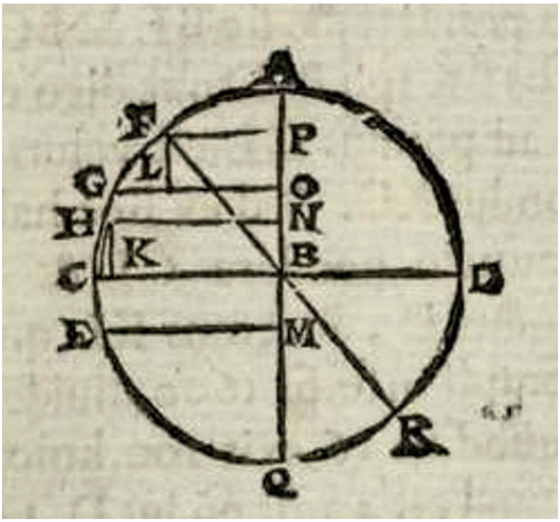

Figure 3.9: According to Cardano, there are three ways to determine positional heaviness. The positional heaviness at point *F*, for instance, may be determined by the horizontal *FP*, by the vertical *FL*, or by the angle *QBF*.

[10]"Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus *QBF* metae, maior est angulo *QBR*. Et similiter quum trutina fuerit *QB*, erit meta *AB*, et tunc angulus *RBA*, maior erit angulo *FBA*, sed maior angulus reddit gravius pondus. […] Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora" (Cardano 1550, 17–18).

3.6.2 Del Monte's Critical Remarks on Positional Heaviness

Del Monte's criticism of Benedetti, in the *Meditatiunculae* as well as in the margin- al remarks of his *Diversae speculationes*, are closely related to his criticism of Nemorarius, Cardano and Tartaglia in the *Mechanicorum liber*. Here, he dealt extensively with the balance and provided a detailed discussion of the theories of these scholars which he deemed to be irremediable. These theories supported the idea that an inclined balance returns to the horizontal and were thus at odds with his own treatment of the matter, which he based on the Archimedean concept of center of gravity. Del Monte believed that an ideal balance would remain in any position as long as it had equal arms, was hinged on its fulcrum and was loaded with equal weights. The only difficulty in testing this theory, he asserted, was the technical difficulty in constructing a perfect balance. It should be noted, moreover, that he assumed that a center of gravity meeting the requirement of his (and Pappus's) definition of center of gravity always exists:

> The center of gravity is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotation by that motion.[11]

Apart from the conceptual irreconcilability between his own approach and that of the Nemorarius school, del Monte tried to demonstrate the inconsistencies of positional heaviness, also within the conceptual framework of his adversaries. One of his main objections was based on a consideration of the cosmological context, which he considered relevant to correctly treat the inclined balance, at least with regards to positional heaviness. Of course, this aspect seems to be relevant when considering Tartaglia's remark that the difference in positional heaviness is infinitesimally small and cannot be compensated by any finite weight resulting from the infinitesimal difference between curvilinear angles.

Contrary to the assumptions of Nemorarius and his successors, del Monte noted that the downward tendencies of the weights are not parallel but converge at the center of the world. Since the directions toward the center of the world from different points on the circular path of the end of the beam cannot be parallel, they are inappropriate for representing positional heaviness. From the fact that those lines converge, he argued further that the lower weight should actually

[11] "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latione circumvertitur" (Monte 1577, 1r). Translation in (Drake and Drabkin 1969, 259), revised in (Renn and Damerow 2010, 57).

become positionally heavier than the higher one. His idea is clearly illustrated by a diagram (Figure 3.10).

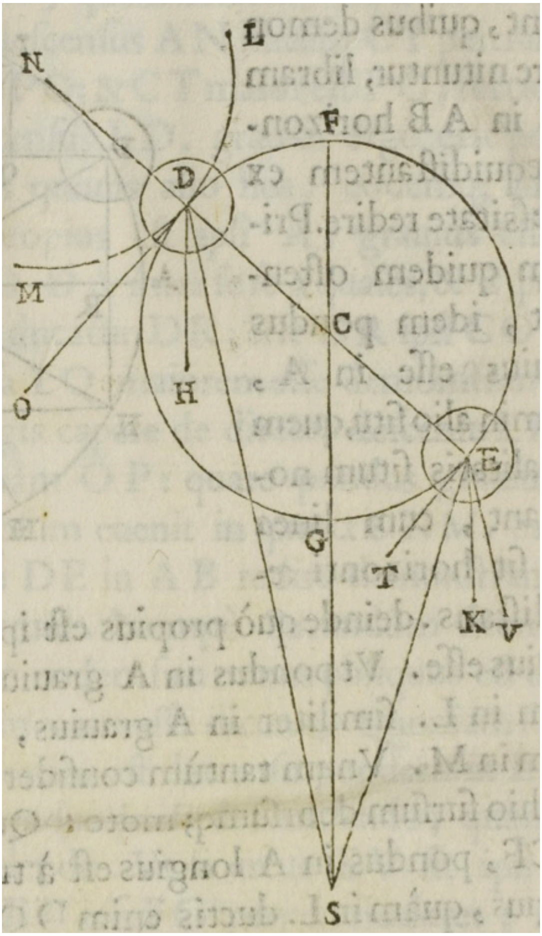

Figure 3.10: According to del Monte, if *S* represents the center of the world, then the mixed angle *SEG* between the circular path of the weight at *E* and the direction to the center of the world is less than the mixed angle *SDG*. Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at *E* must be heavier than that placed at *D*.

Del Monte objected that, from the point of view of positional heaviness, it is not in the horizontal position that a body weighs the most but at that point where a straight line drawn from the center of the world touches at a tangent the circle described by the balance arm. Certainly, if the center of the world were infinitely distant and all lines of direction converging at it were perpendicular and parallel to each other, then the extreme point would mark the horizontal position of the balance arm, also at the fulcrum. Still, for a finite distance from the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum. Del Monte even demonstrated that the closer the balance is to the center of the world, the further this "extreme point" (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum).

Del Monte's crucial objection to the Nemorarius school was that one has not to consider both weights separately, but rather as being connected by the beam of the balance. He drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. According to the positional heaviness, the two weights must be equal. Thus, also from the premises of his adversaries, del Monte could claim that the deflected balance does not return to the horizontal.

3.6.3 Benedetti on Tartaglia's and Nemorarius's Shortcomings

Benedetti confronted the ideas of Tartaglia and Nemorarius on positional heaviness in section seven of his *De mechanicis*. There, Benedetti stressed that by taking into account the distance from the fulcrum to the line of inclination, his approach to the positional effect of a weight was distinct from and superior to Tartaglia's consideration in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central thesis that when a balance is moved from its horizontal position, it will return to this position because the body that has moved upward will attain greater positional heaviness than the body which has moved downward. As we have seen above, Jordanus's and Tartaglia's arguments were based on a comparison of the descents of the two weights. In other words, the balance would have to break in the middle to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by del Monte, that when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and positioned in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he came to the conclusion (correct from a modern viewpoint) that when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

> Therefore the weight of *A* in this [lower] position will be heavier than the weight of *B*.[12]

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line, corresponding to the weight that had been lowered, is longer than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally so that one would expect the balance to tilt into a vertical position.

Benedetti added some more critical remarks on Tartaglia's consideration of positional heaviness. As we have seen, Tartaglia had argued in *Quesiti* that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight. This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument, Benedetti again took into account that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just has del Monte had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even such a small deviation from the parallel must be relevant. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as being parallel to each other.[13]

In summary, Benedetti introduced a way of determining the positional effect of a weight or a force that, in the cases he considered, essentially produces the

[12]"Pondus igitur ipsius *A* in huiusmodi situ, pondere ipsius *B* gravius erit" (Benedetti 1585, 148). Translation in (Drake and Drabkin 1969, 176).

[13]"Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit" (Benedetti 1585, 150). Translation in (Drake and Drabkin 1969, 177).

same results as the application of the modern concept of torque. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical Earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

3.7 Conclusions: The Triangulation Benedetti-del Monte-Galileo

In this chapter, we have dealt with del Monte's and Benedetti's different approaches to mechanics emerging from their reflection on the balance and their treatment of earlier authors. Relative to the issue of positional heaviness, del Monte's self-positioning is essentially external whereas Benedetti positioned himself within the tradition of the Nemorarius school, albeit critically. He explicitly mentioned Tartaglia and Cardano as relevant sources for his treatment, whereas he omitted to mention del Monte (Benedetti 1585, f. A3*r*). In spite of their opposite intentions and mutual suspicion, Benedetti and del Monte shared several opinions and sometimes reached the same conclusions, albeit following different paths: both considered the cosmological center of gravity as relevant for an evaluation (and criticism) of Tartaglia's concept of positional heaviness, and both remarked that one cannot treat the two beams of a balance separately, but rather that they must be considered simultaneously. Moreover, both stressed the ambiguity of the concept of mixed angle and the difficulty of its determination. Nevertheless, their approaches were quite different. As mentioned, Benedetti still worked within the framework of the *gravitas secundum situm*, while del Monte renounced it in favor of the concept of *centrum gravitatis*. For del Monte, the displacement of the balance toward the vertical position was an absurdity that revealed the untenability of Tartaglia's premises. Benedetti deemed this vertical tilt to be the consequence of a correct analysis of the balance based on a conceptuality close to the modern idea of torque, in consideration of the cosmological context. Furthermore, one can stress the importance of Benedetti's attempt to determine the quantity of positional heaviness, a fact that distinguishes him from his predecessors. Additionally, unlike del Monte, he treated the balance by also taking into consideration the general case of forces acting arbitrarily on the beams.

In conclusion, it may be useful to recall the problems linked to the triangulation Benedetti-del Monte-Galileo which might be elucidated by considering the equilibrium controversy. Although the relationship between Benedetti and Galileo is still obscure, the remarkable proximity of these authors on several issues is well known in the history of mechanics. The most recent historical accounts tend in fact to neglect or even deny the possibility of such influence.[14] By contrast, the influence of Benedetti on Galileo was assumed and underscored by earlier scholars like Caverni, Duhem, Wohlwill and Mach (Sarpi 1996).[15] It is helpful to mention the most important issues common to these authors: the attempt at a theory of motion based on Archimedean hydrostatics, the treatment of the acceleration of fall and its causes, the formulation of what in hindsight appear as proto-inertial principles, a similar treatment of the bent lever, the analysis of the relation between vibrating strings and musical tones, their views on the irradiation of surfaces and on thermal and hydrostatic phenomena, and, last but not least, their support of the Copernican world system.[16] Although many of these themes and ideas belonged to the shared knowledge of preclassical mechanics, in some respects the agreement of their approaches is so striking that one may suspect that this is not mere coincidence.[17] Yet, the question of Benedetti's direct impact on Galileo remains unclear, in particular as Benedetti's work was never mentioned by Galileo.

There are a number of possible connections between Benedetti and Galileo that have been considered in the past. For instance, Benedetti is referred to by Galileo's Pisan colleague Jacopo Mazzoni in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597 (Mazzoni 1597). He is often mentioned in the Galileo Studies as the addressee of a famous letter by Galileo arguing for the Copernican system (30 May 1597; Galilei 1968, vol. II, 194–202). In his book, Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous,[18] a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also refers explicitly to Copernicus (Mazzoni 1597, 193; Galilei 1960b, 326). It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni and del Monte during the latter's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.

[14] See the discussion by Ventrice in (Bordiga 1985, 732–736) who mentions Drake, Drabkin, Fredette and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. Notable exceptions are the commentaries by Carugo and Geymonat in their edition of Galileo's *Discorsi* (Carugo and Geymonat 1958). Bertoloni Meli even considers the possibility of del Monte and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on Galileo's thinking because that influence supposedly would have arrived too late, see (Bertoloni Meli 2006, 61–65).
[15] For an overview of such potential connections, see the discussion in (Bordiga 1985, 732–736) who also mentions Mersenne, Clavius and Cardinal Michelangelo Ricci as possible intermediaries.
[16] For an overview, see (Bordiga 1985).
[17] See, for instance, (Drake and Drabkin 1969, 36).
[18] See (Benedetti 1585, 183–184). For a historical discussion of the context of this argument in contemporary technology, see (Freudenthal 2005).

Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*. However, the *Meditatiunculae* may provide the strongest evidence of Galileo's acquaintance with Benedetti's theses.

An important clue is page 145*bis* of the *Meditatiunculae* (Figure 3.11), which is the page opposite the one containing the detailed criticism of Benedetti dealt with in this chapter. This page shows Galileo's construction of the inclined plane reduced to a bent lever.

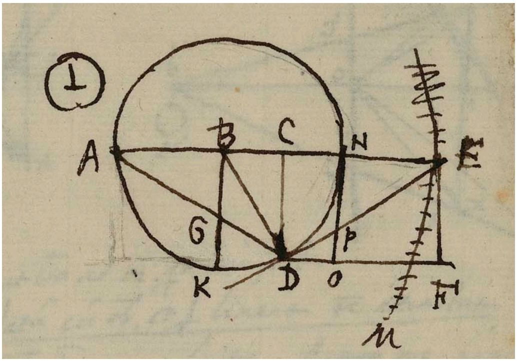

Figure 3.11: Del Monte, *Meditatiunculae*, p. 145*bis* showing Galileo's construction relating the bent lever to the inclined plane.

This fact is all the more noteworthy since del Monte's notebook, on an earlier page, also contains his own problematic adoption of Pappus's analysis of the inclined plane (Monte 1587, 64. Figure 3.12). In his writings, Galileo had criticized this analysis, substituting it with his own solution of the problem which makes use of the bent lever conceptualized in the same way as Benedetti (Galilei 1960a, 172). Del Monte therefore must have learned about this proof from Galileo, and he must also have seen the connection to Benedetti's methods. In any case, it is likely that the two scientists discussed this connection and quite plausible that Galileo became familiar with Benedetti's work through del Monte. Galileo began to correspond with del Monte in 1588, three years after the publication of Benedetti's *Diversae speculationes* and shortly before he embarked on the writings that later became known as *De Motu* (Galilei 1960b).[19] Galileo first wrote a dialogue version of *De Motu* and then an essay in twenty-three chapters. Only the second essay version of these writings contains his proof of the law of the

[19]For a thorough discussion of the chronology of these writings, see (Giusti 1998).

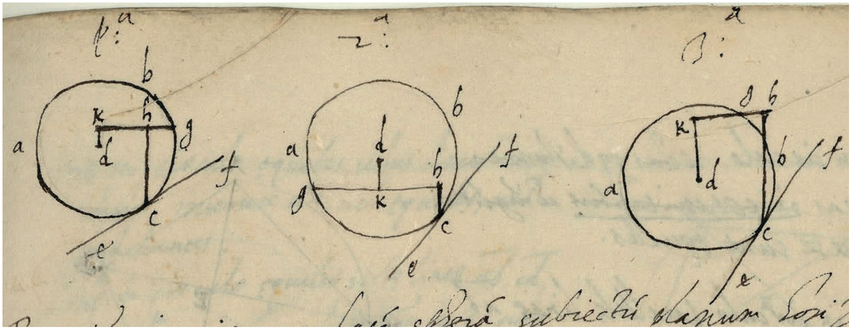

Figure 3.12: Del Monte's construction related to the inclined plane on p. 64 of his notebook. The construction was adapted from Pappus's erroneous solution.

inclined plane, the argument about continuity of motion along a straight line, and a mention of Copernicus. This version was most likely written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time discussed explicitly the problem of the effective lever arm, was written much later, certainly after he had visited del Monte in 1592 during his journey to Padua. Hence, it seems most likely that Galileo was already familiar with key ideas of Benedetti at the time of writing these works.

Recent research into del Monte's biography has shown that del Monte and Galileo must have met as early as 1589 in Tuscany (see Menchetti's contribution in this volume). They might even have met jointly with Galileo's teacher, Mazzoni who, as mentioned earlier, cited Benedetti in his work. Thus, del Monte, Mazzoni and Galileo may have discussed Benedetti's *Diversarum speculationum ... liber*, leading Galileo to reconsider his work in progress on motion and, in particular, his treatment of motion along inclined planes, making use of Benedetti's theory of the bent lever that was mentioned in del Monte's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may now have taken the Copernican hypothesis much more seriously than before, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597, Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they had meanwhile reached an agreement, trying now also to press him on the Copernican hypothesis.[20]

[20]This scenario was developed in a joint discussion with Pier Daniele Napolitani. Concerning Benedetti's adherence to the Copernican system, see (Di Bono 1987; Omodeo 2009).

In particular, Galileo's concept of *momento*[21] and his analysis of the bent lever—crucial to both his mechanics and his theory of motion—evidently emerged from the midst of the controversy about positional heaviness. In that debate, Galileo took a position much closer to Benedetti than to del Monte. Rather than *gravitas secundum situm*, Galileo used the concept of *momento* or *momentum* that del Monte had introduced in his book by quoting Commandino's definition of the center of gravity. But while del Monte made no further use of this in his mechanics, Galileo took this concept from the respected Urbino school, gave it a new meaning that was taken from Benedetti and made it a pillar of his own conception, which included Commandino's definition of the center of gravity:

> Center of gravity is defined to be that point in every heavy body around which parts of equal moments are arranged.[22]

The evidence for this claim concerning Benedetti's legacy in Galileo's work derives from the marginal notes del Monte made in his copy of Benedetti's book, as well as from his entries in the *Meditatiunculae* (Monte 1587) which contain traces of Galileo's intervention in this controversy.

According to Benedetti and Galileo (and contrary to Tartaglia and del Monte), the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum of the balance to the line of inclination, determines the effectiveness of a weight or a mechanical constellation. In his *Mechanics*, Galileo later stressed how important it is to carefully define the effective distances of weights from their support:

> There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.[23]

In his analysis of the inclined plane using the bent lever, Galileo also made clear that this procedure is critical for determining the *momento* of a given weight

[21] See the extensive discussion in (Galluzzi 1979).

[22] "Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti" (Galilei 1968, vol. 2, 159). Translation in (Galilei 1960a, 151). See also (Galilei 2002).

[23] "Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi" (Galilei 1968, vol. 2, 164). Translation in (Galilei 1960a, 156–157).

(Galilei 1968, vol. 2, 181; Galilei 1960a, 173). As discussed earlier, in his *Diversarum speculationum [...] liber* Benedetti convincingly demonstrated the efficacy of this method for determining the magnitude of a force or weight according to its position.

In conclusion, the very existence of del Monte's annotations on his copy of Benedetti's *Diversarum speculationum [...] liber* provides a definitive answer to the question of who actually read this book.[24] It is also difficult to imagine that he did not discuss his views on Benedetti's mechanics with Galileo, views that he considered both misguided and profoundly challenging, as is made evident in his handwritten notes. It was most probably del Monte, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo. At the same time, he also made it virtually impossible for Galileo to openly admit to Benedetti's influence if did he not want to jeopardize the protection of this most important patron of his early career.

3.8 Appendix 1: Benedetti's *De Mechanicis*, Chapters I–III

DE MECHANICIS

Scripserunt multi multa, et quidem scitissime, de mechanicis, at cum natura ususque aliquid semper vel novum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posteris inuidere, si quid ei contigerit comperuisse prius tenebris involutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quaedam futura, ut reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exacte explicata in medium proferre volui: quo vel iuvandi desiderium, vel saltem non ociosi ingenioli argumentum aliquod exhiberem: atque vel hoc uno modo me inter humanos vixisse testatum relinquerem.

3.8.1 De differentia situs brachiorum librae. CAP. I.

Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii. Sit exempli gratia *B* centrum, aut, quod dividit brachia alicuius librae, et *ABQ* verticalis linea, aut, ut rectius dicam, axis orizontis, et *BC* unum brachium dictae librae, et in *C* sit pondus, et *CO* linea inclinationis, seu itineris *C* versus centrum mundi, cum qua *BC* angulum rectum constituat in puncto *C*. Existente igitur in huiusmodi situ brachio *BC* dico pondus *C* gravius futurum, quam in alio quolibet situ quia supra centrum *B* omnino non quiescet, quemadmodum in quovis alio situ faceret.

[24] The knowledge that he had read it, however, is not entirely new. See (Renn, Damerow, and Rieger 2001, 74).

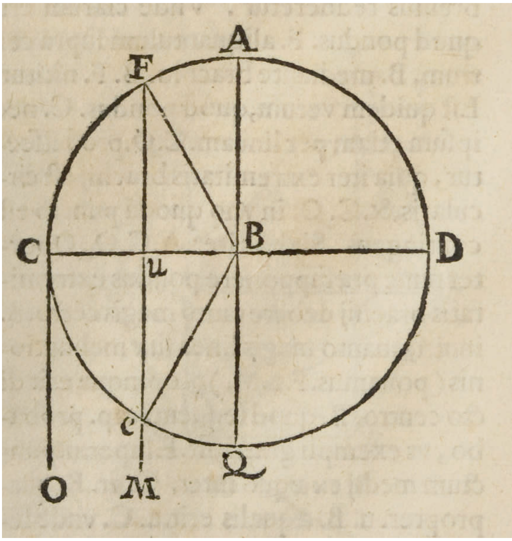

Figure 3.13

Ad quod intelligendum, sit dictum brachium, in situ *BF* cum eodem pondere in puncto *F* et linea itineris seu inclinationis dicti ponderis sit *FUM* per quam lineam dictum pondus progredi non potest, nisi brachium *BF* breuius redderetur. Unde clarum erit quod pondus *F* aliquantulum supra centrum *B* mediante brachio *BF* nititur. Est quidem verum, quod pondus *C* nec ipsum etiam per lineam *CO* proficiscetur, quia iter extremitatis brachii est circularis, et *CO* in uno quodam puncto est contingens. Sit hoc iter *ACQ* oportet nunc praesupponere pondus extremitatis brachii debere tanto magis centro *B* inniti, quanto magis linea suae inclinationis (ponamus *FUM*) propinqua erit dicto centro *B* quod sequenti cap. probabo, ut exempli gratia, sit *F* super *U* punctum medii ex aequo inter *C* et *B* quapropter *UB* aequalis erit *UC* unde sequetur dictum pondus gravius futurum pro parte *FC* quam pro ea, quae est *AF* et minus supra centrum *B* pro dicta parte *FC* quam pro parte *AF* quieturum; et dictum brachium quanto magis orizontale erit a situ *BF* tanto minus supra dictum centrum *B* quiescet, et hac ratione gravius quoque erit, et quanto magis vicinum erit ipsi *A* a dicto *F* tanto magis super centrum *B* quoque quiescet, unde tanto quoque levius existet. Idem dico de omni situ brachii per girum inferiorem *CQ* ubi pondus pendebit a centro *B* dictum centrum attrahendo, quemadmodum superius illud impellebat. Haec vero omnia cap. sequenti melius percipientur.

3.8.2 De proportione ponderis extremitatis brachii librae in diverso situ ab orizontali. CAP. II.

Proportio ponderis in *C* ad idem pondus in *F* erit quemadmodum totius brachii *BC* ad partem *BU* positam inter centrum et lineam *FUM* inclinationis, quam pondus ab extremitate *F* liberum versus mundi centrum conficeret. Quod ut facilius intelligamus imaginemur alterum brachium librae *BD* et in extremo *D* locatum aliquod pondus minus pondere *C* ut *BU* pars *BC* minor est *BD*. Clare cognoscetur ex 6 lib. primi *de ponderibus* Archimedis, quod si in puncto *U* collocatum erit pondus ipsius *C* libra nihil penitus a situ orizontali dimovebitur. Sed perinde est quod pondus *F* aequale *C* sit in extremo *F* in situ brachii *BF* quam ut sit in puncto *U* in situ ipsius *BU* orizontali. Ad cuius rei evidentiam imaginemur filum, *FU* perpendiculare, et in cuius extremo *U* pendere pondus, quod erat in *F* unde clarum erit quod eundem effectum gignet, ac si fuisset in *F* quod, ut iam diximus remanens affixum puncto *U* brachii *BU* tanto minus grave est situ ipsius *C* quanto *UB* minus est ipso *BC*. Idem assero si brachium esset in situ *EB* quod facile cognoscere poterimus, si imaginemur filum appensum ipsi *U* brachii *BC* et usque ad *E* perpendicularem, in quo extremo appensum esset pondus aequale ponderi *C* et liberum ab *E* brachii *BE* unde libra orizontalis manebit. Sed si brachium *BE* consolidatum fuisset in tali situ cum orizontali *BD* et appenso pondere *C* in *E* libero a filo, nec ascenderet, neque descenderet, quia tantum est quod ipsum sit appensum filo, quod pendet ab *U* quantum quod ab ipso liberum appensum fuisset *E* brachii *BE* et hoc procederet ab eo quod partim penderet a centro *B* et si brachium esset in situ *BQ* totum pondus centro *B* remaneret appensum, quemadmodum in situ *BA* totum dicto centro anniteretur. Unde fit ut hoc modo pondus magis aut minus sit grave, quo magis aut minus a centro pendet, aut eidem nititur: atque haec est causa proxima, et per se, qua fit ut unum idemque pondus in uno eodemque medio magis aut minus grave existat. Et quamvis appellem latus *BC* orizontale, supponens illud angulum rectum cum *CO* facere, unde angulus *CBQ* fit ut minor sit recto, ob quantitatem unius anguli aequalis ei, quem duae *CO* et *BQ* in centro regionis elementaris constituunt, hoc tamen nihil refert, cum dictus angulus insensibilis sit magnitudinis. Ab istis autem rationibus elicere possumus, quod si punctus *U* erit ex aequo medius inter centrum *B* et extremum *C* pondus *F* aut *M* pendebit, aut nitetur pro medietate dicto centro *B* et si dictum *U* erit propius *B* quam puncto *C* pendebit ab ipso, aut nitetur ipsi amplius quam ex medietate, et si magis versus *C* minus quam ex medietate nitetur.

3.8.3 Quod quantitas cuiuslibet ponderis, aut virtus movens respectu alterius quantitatis cognoscatur beneficio perpendicularium ductarum a centro librae ad lineam inclinationis. CAP. III.

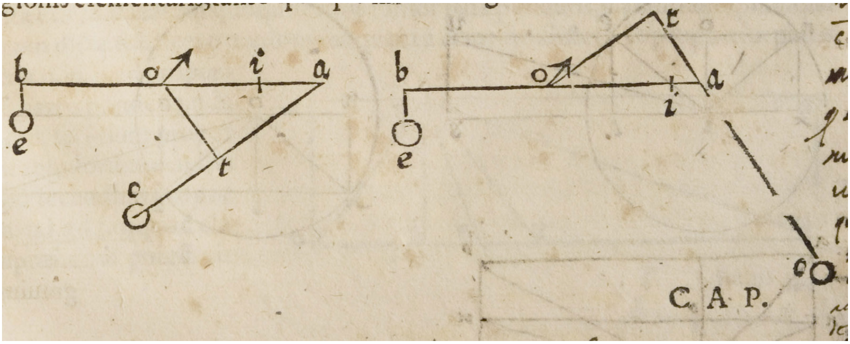

Figure 3.14

Ex iis, quae a nobis hucusque sunt dicta, facile intelligi potest, quod quantitas *BU* quae fere perpendicularis est a centro *B* ad lineam *FU* inclinationis, ea est, quae nos ducit in cognitionem quantitatis virtutis ipsius *F* in huiusmodi situ, constituens videlicet linea *FU* cum brachio *FB* angulum acutum *BFU*. Ut hoc tamen melius intelligamus, imaginemur libram *BOA* fixam in centro *O* ad cuius etrema sint appensa duo pondera, aut duae virtutes moventes *E* et *C* ita tamen quod linea inclinationis *E* id est *BE* faciat angulum rectum cum *OB* in puncto *B*. Linea vero inclinationis *C* id est *AC* faciat angulum acutum, aut obtusum cum *OA* in puncto *A*. Imaginemur ergo lineam *OT* perpendicularem lineae *CA* inclinationis, unde *OT* minor erit *OA* ex 18 primi Euclidis. Secetur deinde imaginatione *OA* in puncto *I* ita ut *OI* aequalis sit *OT* et puncto *I* appensum sit pondus aequale ipsi *C* cuius inclinationis linea parallela sit lineae inclinationis ponderis *E* supponendo tamen pondus aut virtutem *C* ea ratione maiorem esse ea, quae est *E* qua *BO* maior est *OT* absque dubio ex 6. lib. primi Archi. *De ponderibus BOI* non movebitur situ, sed si loco *OI* imaginabimur *OT* consolidatam cum *OB* et per lineam *TC* attractam virtute *C* similiter quoque continget ut *BOT*, communi quadam scientia, non moveatur situ. Est ergo quod proposuimus verum quantitatem alicuius ponderis respectu ad eam, quae est alterius debere depraehendi a perpendicularibus, quae a centro librae ad lineas inclinationis exiliunt. Hinc autem innotescit facillime, quantum vigoris, et vis pondus, aut virtus *C* ad angulum rectum cum *OA* minime trahens, amitttat. Hinc quoque corollarium quoddam sequetur, quod quanto propinquius erit centrum *O* librae centro regionis elementaris, tanto quoque minus erit grave.

3.9 Appendix 2: Del Monte's *Meditatiunculae*, ff. 145 and 146

Tassora's transcription has been slightly revised here.

3.9.1 Contra Cap. 2 Jo. de Benedicti de Mechanicis

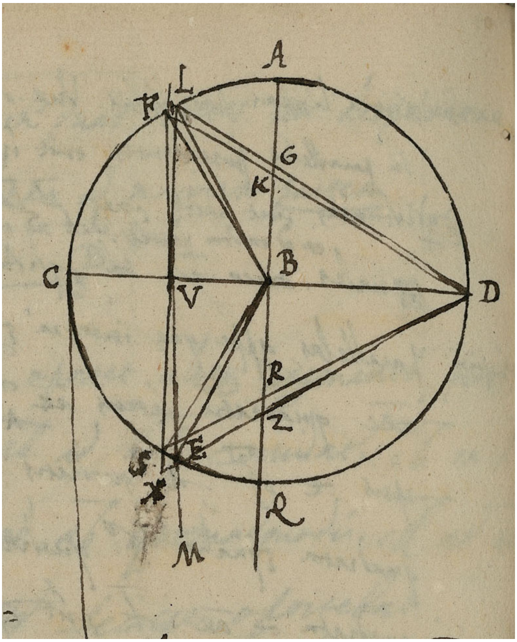

Figure 3.15

Inquit auctor in demonstratione idem pondus in *F*, aeque grave esse ut in *U* et in *E*. Quod est tamen falsum.

Nam lineae *FM AQ* non sunt aequidistantes, cum in centrum mundi conveniant.

Ac propterea ducta per *U* linea *LUS* ipsi *AQ* aequidistante; erit *UL* inter *FU AB*; *UE* vero inter *US* et *BQ*.

Quare ducta *SRD*, erit *BD* ad *BU*, ut *DR* ad *RS*. Ac propterea si *BU* dimidia est ipsius *BD*, et *SR* erit dimidia ipsius *RD*.

Si igitur ducatur *BS*, quae intelligatur consolidata cum *BD* ponaturque pondus in *S* duplum ponderis *D*, aequeponderabunt pondera *SD* ex distantiis *BD RS* ita constitutis, cum sit *R* ipsorum centrum gravitatis in linea *BQ*. Hoc est in infimo loco. Ut ex nostris mechanicis patet.

Pondus igitur in *S* aequegrave erit, atque *U* non autem pondus in *E*, ut ipse existimat. Idem enim pondus gravius est in *S* quam in *E*.

Ut ipse fatetur quod probabitur quoque hoc modo. Nam productis *LS DE* in *X* est quidem *DZ* ad *ZX*, ut *DR* ad *RS*. Atque maiorem habet proportionem *DZ* ad *ZE*, quam ad *ZX*; duplum igitur ponderis *D* in *X* ipsi *D* aequeponderabit. Positum ergo in *E* ipsi *D* non aequeponderabit. Et ut aequeponderet, maius erit quam duplum.

Similiter ad partem *F* ducta *LGB* quoniam *LU* est *GB* aequidistans; erit *DG* ad *GL*, ut *DB* ad *BU*. Si igitur intelligatur *BL* consolidata cum *BD*, idem pondus, tam in *L*, quam in *U* eidem ponderi in *D* aequeponderabit, cum *G* sit centrum gravitatis ponderum in *L D* existentium. Non igitur pondus in *F* aequegrave est, ut idem pondus in *U*.

Praeterea secet *FD* ipsam *LU* in *H*. Patet idem pondus in *U* et in *H* ipsi ponderi in *D* aequeponderare. Cum sit *DK* ad *KH*, ut *DB* ad *BU*, et *DG* ad *GL*. Minorem autem proportionem habet *DK* ad *KF*, quam ad *KH*. Minus igitur pondus in *F* quam duplum ipsius *D*, ipsi *D* aequeponderabit.

Et quibus etiam constat idem pondus in *F*, et in *U*, et in *E*, diversi modo gravitare. Gravius est enim in situ *E* quam in *U* et in *F*. In *U* vero gravius, quam in *F*.

Fallacia vero argumenti est cum inquit, existente filo *FUE* perpendiculari, idem pondus in *F* et in *U* eodem modo gravitabit. Quod est quidem verum, si intelligatur quod eodem modo gravitet in *F* a quo libere pendet.

Cum vero inquit, quoniam punctum fili *U* secet *BC* in *U*, ergo pondus in puncto *U* librae *DBU*, ac propterea in *U* brachii *BU* eandem habebit gravitatem ut in *F*; est falsum. <Nunc> enim valet consequentia pondus in filo in *U* eandem habet gravitatem ut in *F*. Ergo pondus in *U* brachii *BU* eandem habet gravitatem ut in *F*. Veluti quoque falsum est propter filum pondus in *E* est aequegrave, ut pondus in *U* brachii *BU*. Non est igitur haec vera et proxima causa, et per se harum gravitatum. Ut ipse profitetur.

3.9.2 Against Chapter 2 of Giovanni Benedetti's [treatise] on Mechanics

The author claims in his proof that the same weight in *F* is equally heavy as in *U* and in *E*, but this is false.

The lines *FM AQ* are namely not equally distant, because they converge in the center of the world.

And therefore if the line *LUS* is drawn through *U* equidistant to *AQ*, *UL* will be between *FU AB*, but *UE* between *US* and *BQ*.

For if one draws *SRD*, *BD* will be to *BU*, as *DR* to *RS*. And hence if *BU* is half of the same *BD*, also *SR* will be the half of the same *RD*.

If therefore *BS* is drawn, which shall be understood as being rigidly connected with *BD*, and if a weight is placed in *S* which is double the weight [in] *D*, the weights *SD* will be in equilibrium from the distances *BD RS* thus constituted, because their center of gravity *R* is in the line *BQ*; that is in the lowest place; as is evident from our [book on] mechanics.

The weight in *S* will therefore be equally heavy as [the weight in] *U* but not as the weight in *E* as he believes. The same weight is namely more heavy in *S* than in *E*.

As he himself admits this can also be proven in the following way. Because when *LS DE* are prolongued [to meet] in *X*, then evidently *DZ* is to *ZX* as is *DR* to *RS*. But *DZ* has a larger proportion to *ZE* than to *ZX*; the double of the weight *D* in *X* will therefore be equally heavy as the same *D*. Hence placed in *E* it will not be equally heavy as *D*. And if it were in equilibrium, it would be more than double.

Similarly let *LGB* be drawn to *F* being *LU* equidistant to *GB*; *DG* will be to *GL* as *DB* to *BU*. If now *BL* is understood as being connected with *BD*, the same weight, in *L* as in *U* will be in equilibrium with the same weight in *D*, because *G* is the center of gravity of the weights existing in *L* and *D*. Therefore the weight in *F* is not equally heavy as the same weight in *U*.

Let furthermore *FD* cut the same *LU* in *H*. It is clear that the same weight in *U* and in *H* will be equally heavy with regard to the same weight in *D*. Because *DK* is to *KB* as is *DB* to *BN* and as *DG* to *GL*. But *DK* has a smaller proportion to *KF* as to *KH*. Therefore in *F* a weight smaller than double [the weight in] *D* will be in equilibrium with the same weight *D*.

From this it is also clear that the weight in *F*, in *U*, and in *E* gravitates in a different ways. It is namely heavier in the position *E* than it is in *U* and in *F*. But in *U* it is heavier than in *F*.

But the fallacy of the argument emerges when he says that, being the thread *FUE* perpendicular, the weight in *F* has the same heaviness as in *U*. What is indeed true if it is understood that it gravitates in the same way in *F* from which it freely hangs.

But if he says, because the point of the thread *U* cuts *BC* in *U*, therefore the weight in the point *U* of the balance *DBU* will hence have the same heaviness in *U* of the lever arm *BU* as in *F*, then this is false. Now the consequence holds that the weight on the thread in *U* has the same heaviness as in *F*. Thus, the weight in *U* of the beam *BU* has the same heaviness as in *F*. In the same way it is also false that because of the thread the weight in *E* is equally heavy as the weight in *U* of the arm *BU*. This is therefore not the true and next cause, and [the cause] itself [*per se*] for them of these heaviness [proportions], contrary to that which he contends.

3.9.3 Contra <capitulum> 3 eiusdem

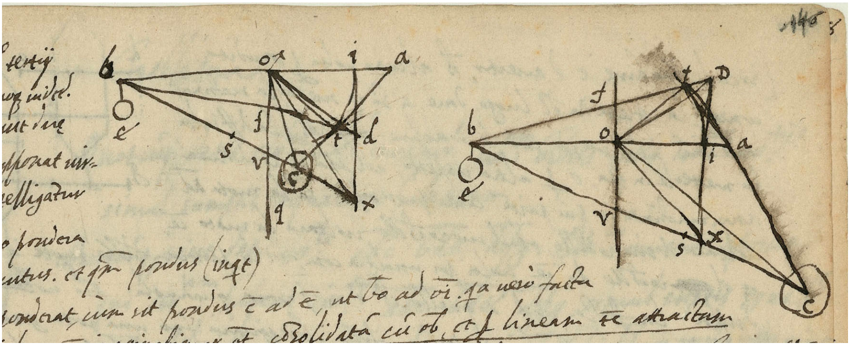

Figure 3.16

Falsum est igitur ex dictis, quod in principio tertii <capitoli> inquit. Praeterea demonstratio falsa quoque videtur.

Inquit enim sint *E C* duo pondera, aut duae virtutes, ita ut intelligat, et supponat virtutes ponderum officio fungi. Intelligantur itaque ad maiorem evidentia duo pondera *E C*. Sitque *BAC* angulus primum acutus.

Et quoniam pondus (inquit) in *I* aequale *C* ipsi <*E*> aequeponderat, cum sit pondus *C* ad pondus *E*, ut *BO* ad *OI*. Quia vero facta est *OI* aequalis *OT* inquit.

Si loco *OI* imaginabimur *OT* consolidata cum *OB*, et per lineam *TC* attractam virtute *C*, similiter quoque continget, ut *BOT*, communi quadam scientia, non moveatur situ.

Fateor me hanc quamdam communem scientiam non intelligere.

At perpendamus sensum quod nil aliud significat, nisi quod idem pondus ipsi *C* aequale, in *I*, rectam libram *BOI*, idemque pondus *C* consolidatam libram *BOTC*, ponderi *E* aequeponderat. Quod esse non potest.

Nam si intelligatur linea *BA* horizonti aequidistans. Centroque *O* circulus describatur *IT*, idem pondus gravius erit in *I*, quam in *T*.

Quare pondus in *T* ipsi *C* aequale non aequeponderabit libram *TOB*.

Quod patet etiam ducta primum *OQ* linea perpendiculari, quam ipse lineam verticalem, et axem horizontis nuncupat. Deinde ducatur *ID* ipsi *OQ* aequidistans, ducaturque *BFTD*: erit *BF* ad *FD*, ut *BO* ad *OI*. Si igitur intelligatur *OD* consolidata cum *OB*, idem pondus in *D* ipsi *E* aequeponderabit, cum punctum *F* ponderum in *BD* centrum gravitatis <existens> sit in linea *OFQ*. Pondus ergo in *T* ipsi *E* non aequeponderabit. Multoque minus pondus *C* ipsi *E* aequeponderare potest.

Nam si iungatur *BC*, fiatque ut *C* ad *E*, ita *BS* ad *SC*; erit *S* ponderum centrum gravitatis. Quod quidem in linea *OQ* existere non potest.

Productis enim *ID BC* in *X*; erit *BO* ad *OI*, ut *BU* ad *UX*. Quare ducta *OX*, quae intelligatur consolidata cum *BO*, pondus in *X* aequale ipsi *C* ponderi *E* aequeponderabit. Itaque existente pondere *C* in recta linea *BCX*, intelligaturque ducta *CO* consolidata cum *OB*; pondus *C* non aequeponderabit *E*.

Idem enim sequitur sive intelliganturque *CO OB* consolidatae, sive *CT TO OB* consolidatae. Non enim punctum *U* esse potest centrum gravitatis ponderum in *B C* existentium. Cum maiorem habeat proportionem *BU* ad *UC* quam ad *UX*, ac propterea maiorem quam pondus *C* ad *E*. Quare centrum gravitatis *S* ponderum in *CB* est inter *UB*. Numquam autem manebit libra *COTB*, donec punctum *S* sit in linea *OQ*. Ergo non aequeponderabunt.

Similiter existente *BAC* angulo obtuso, ostendetur pondus in *T* minorem habere gravitatem, quam in *I*.

Deinde pondus in *X* aequale ipsi *C* aequeponderare ipsi *E*; cum sit *BU* ad *UX*, ut *BO* ad *OI*.

Si itaque sit *S* centrum gravitatis ponderum in *B C*; erit *S* inter *UC*. Quare cum non sit *S* in linea *OQ*. Pondera *C* e consolidatam libram *CTOB* non aequeposerabunt.

Falsa igitur est demonstratio. Fallacia vero est, cum inquit, continget, ut *BOT* communi quadam scientia, non <moveatur> situ.

Et est omnino falsum si intelligatur *C* esse pondus, quod in centrum mundi sempre tendit. Ut ipse supponere videtur. Et ut ipse in seguentibus <capitolis> accipit hoc tamquam de ponderibus demonstratum.

At vero si intelligatur *I* potentia movens, ut hominis, qui potest trahere *T* per rectam lineam *TC*, tunc vera esse potest demonstratio. Ut patet ex tractatum de axe in peritrochio nostrorum Mechanicorum.

Notandum tamen, quod conclusiones per communem quandam scientiam deductae, non sunt periti mathematici cum propriis uti oporteat.

Ex hac etiam figura magis patet absurdum, hoc est pondera *E C* aequeponderare non posse.

3.9.4 In Opposition to Chapter 3

It is therefore false, from what has been said, what he says in the beginning of the third chapter. Moreover also the demonstration seems to be wrong.

He says namely that *E* and *C* are two weights, or two forces, so that he understands and assumes that the forces take over the role of the weights. Let therefore, for major clarity, *E* and *C* be understood to be two weights. And let *BAC* first be an acute angle.

And since (he says) the weight in *I* equal to *C* will be equally heavy to that in <*E*>, because the weight *C* is to the weight *E* as is *BO* to *OI*. Because *OI* is made equal to *OT* he says.

If we shall imagine instead of *OI OT* to be rigidly connected with *OB*, and along the line *TC* attracted by the force *C*, it is also similarly the case that *BOT*, by a certain common science, will not change place.

I admit that I do not understand this certain common science.

We guess that this means nothing else but that the same weight, equal to *C*, in *I*, by the straight balance *BOI*, and the same weight *C*, if the balance *BOTC* is [conceived to be] solid, will be equally heavy to the weight *E*. Which cannot be.

For if it is understood that the line *BA* is equidistant from the horizon, and a circle *IT* is described with center *O*, the same weight will be heavier in *I* than in *T*.

Because the weight in *T*, equal to that [in] *C*, will not be in equilibrium with the balance *TOB*.

This is also evident when one first draws the line *OQ* perpendicularly, which he calls vertical line and axis of the horizon. Then let *ID* be drawn equidistant to *OQ*, and let *BFTD* be drawn: then *BF* will be to *FD*, as *BO* to *OI*. If therefore *OD* is understood as being rigidly connected with *OB*, the same weight in *D* will be in equilibrium with the same *E*, because the point *F*, the center of gravity of the weights in *BD*, is in the line *OFQ*; hence the weight in *T* is not in equilibrium with the weight *E*. And a much less smaller weight *C* can be in equilibrium with the same *E*.

For if *BC* is connected, and if we let as *C* to *E*, be *BS* ad *SC*, then *S* will be the center of gravity of the weights. This [center], however, cannot exist in the line *OQ*.

If namely *ID BC* are prolongued [to meet] in *X*; then *BO* will be to *OI* as *BU* to *UX*. For which reason if *OX* is drawn, which is understood as being rigidly connected with *BO*, the weight in *X*, equal to the same *C* will be in equilibrium with the weight *E*. Therefore, if the weight *C* exists in the straight line *BCX*, and if it is understood that *CO* is drawn [and] rigidly connected with *OB*, the weight *C* will not be in equilibrium with *E*.

The same follows namely when alternatively *CO* and *OB* are understood to be rigidly connected, or when *CT*, *TO*, and *OB* are connected. For the point *U* cannot be the center of gravity of the weights existing in *B C*. Because *BU* has to *UC* a larger proportion than to *UX*, and hence a major [proportion] than the weight *C* to *E*. Because the center of gravity *S* of the weights in *CB* is between *UB*. But the balance *COTB* will never remain, as long as the point *S* is in the line *OQ*. Therefore they will not be in equilibrium.

Similarly, if there is an obtuse angle *BAC*, it is shown that the weight in *T* has a smaller heaviness than in *I*.

Then the weight in *X* equal to the same *C* [is claimed] to be in equilibrium to the same *E*; because *BU* is to *UX*, as is BO to *OI*.

If therefore *S* is the center of gravity of the weights in *B C*, *S* will be between *UC*. For which reason because *S* is not in the line *OQ*, the weights *C* and the rigidly connected balance *CTOB* will not be in equilibrium.

The demonstration is therefore false. Actually, the fallacy is [to say] that *BOT*, according to some common science, does not <change> its place.

And it is totally false if *C* is understood to be a weight which always tends to the center of the world as he seems to assume and as he in the following <chapters> assumes it to be demonstrated as if it holds for weights.

To speak the truth, if *I* is understood to be a moving power, like that of a man who can draw *T* along the straight line *TC*, then the demonstration can be true. In fact, it is clear from our treatise on the axis on the wheel [*de axe in peritrochio*] of our *Mechanics*.

It nevertheless has to be noted that the conclusions which are inferred by a certain common science should not be used by an experienced mathematician because he should use his own.

From this figure descends an even greater absurdity, namely that the weights *E C* cannot be in equilibrium.

References

Benedetti, G. B. (1585). *Diversarum speculationum mathematicarum, et physicarum liber: quarum seriem sequens pagina indicabit*. Torino: Bevilaqua.

Bertoloni Meli, D. (2006). *Thinking with Objects: The Transformation of Mechanics in the Seventeenth Century*. Baltimore: Johns Hopkins University Press.

Bordiga, G. (1985). *Giovanni Battista Benedetti: filosofo e matematico veneziano del secolo XVI. Con un aggiornamento bibliografico ragionato di Pasquale Ventrice*. Venezia: Istituto Veneto di Scienze, Lettere ed Arti.

Cardano, G. (1550). *Hieronymi Cardani Medici Mediolanensis de subtilitate libri XXI*. Nürnberg: Petreius.

Carugo, A. and L. Geymonat (1958). *Galieo Galilei: Discorsi e dimostrazioni matematiche intorno a due nuove scienze*. Torino: Boringhieri.

Di Bono, M. (1987). L'astronomia copernicana nell'opera di Giovan Battista Benedetti. In: *Cultura, scienze e tecniche nella Venezia del Cinquecento*. Venezia: Istituto Veneto di Scienze, Lettere ed Arti, 283–300.

Drake, S. and I. E. Drabkin (1969). *Mechanics in Sixteenth-Century Italy*. Madison: University of Wisconsin Press.

Freudenthal, G. (2005). The Hessen-Grossman Thesis: An Attempt at Rehabilitation. *Perspectives on Science* 13(2):166–193.

Galilei, G. (1960a). On Mechanics. Translated with Introduction and Notes by Stillman Drake. In: *On Motion and on Mechanics*. Ed. by I. E. Drabkin and S. Drake. Madison: The University of Wisconsin Press, 133–186.

— (1960b). On Motion. Translated with an Introduction and Notes by I. E. Drabkin. In: *On Motion and on Mechanics*. Ed. by I. E. Drabkin and S. Drake. Madison: The University of Wisconsin Press, 1–131.

— (1968). *G. Galilei, Le opere*. Ed. by A. Favaro. Florence: Barbera.

— (2002). *Le mecaniche*. Ed. by R. Gatto. Firenze: Olschki.

Galluzzi, P. (1979). *Momento. Studi galileiani*. Roma: Ed. dell'Ateneo & Bizzari.

Giusti, E. (1998). Elements for the Relative Chronology of Galilei's "De motu antiquiora". *Nuncius* XIII:427–460.

Hispanus, P. (1972). *Tractatus sive summule logicales*. Ed. by Ch. Rapp and K. Corcilius. Assen: van Gorcum.

Mazzoni, J. (1597). *In universam Platonis et Aristotelis philosophiam praeludia*. Venezia: Guerilius.

Monte, Guidobaldo del (1577). *Mechanicorum liber*. Pesaro: Hieronymum Concordiam.

— (1587). *Meditantiunculae Guidi Ubaldi ex marchionibus montis Santae Mariae de rebus mathematicis (ca. 1587-1592)*. Bibliothèque Nationale de Paris, manuscript, catalogue no. Lat. 10246.

Nemorarius, J. (1533). *Liber Jordani Nemorarii viri clarissimi de ponderibus proportiones XIII et earundem demonstrationes, multarumque rerum rationes sane pulcherrimas complectens, nunc in lucem editus cum gratia et privilegio imperiali, Petro Apiano mathematico ingolstadiano ad xxx. annos concesso*. Nürnberg: Petreius.

— (1565). *Jordani opusculum de ponderositate Nicolai Tartaleae studio correctum, novisque figuris auctum*. Venezia: Curzio Troiano Navo.

Omodeo, P. D. (2009). La cosmologia infinitistica di Giovanni Battista Benedetti. *Bruniana & Campanelliana* 15(1):181–190.

Renn, J. and P. Damerow (2010). *Guidobaldo del Monte's Mechanicorum liber*. Berlin: Edition Open Access.

— (2012). *The Equilibrium Controversy: Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Background*. Berlin: Edition Open Access.

Renn, J., P. Damerow, and S. Rieger (2001). Hunting the White Elephant. When and How Did Galileo Discover the Law of Fall? In: *Galileo in Context*. Ed. by J. Renn. Cambridge: Cambridge University Press, 29–149.

Sarpi, P. (1996). *Pensieri naturali, metafisici e matematici*. Ed. by L. Cozzi and L. Sosio. Milano: Ricciardi.

Tassora, R. (2001). *Le Meditatiunculae de Rebus Mathematicis di Guidobaldo del Monte*. Ph. D. thesis. Università di Bari.