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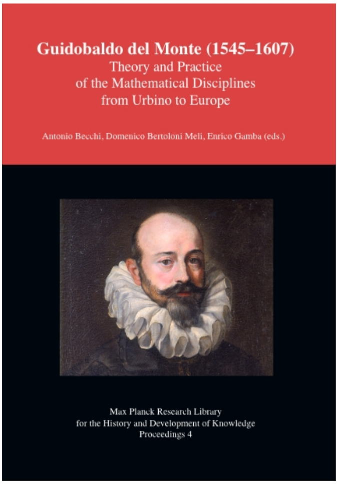
Domenico Bertoloni Meli:

Guidobaldo, Galileo, and the History of Mechanics



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Chapter 5
Guidobaldo, Galileo, and the History of Mechanics

Domenico Bertoloni Meli

5.1 Introduction

In this essay I consider some perspectives from which Guidobaldo del Monte's work on mechanics was viewed in the historical literature around his time and in subsequent centuries. At first sight it may seem peculiar to address the matter from this angle, since from the greatest part of the current historiography Guidobaldo's chief contribution to mechanics—the 1577 *Mechanicorum liber*—appears at best largely unrelated and at worst a hindrance to the tumultuous developments of this field, especially the science of motion right to the time of Isaac Newton and the birth of analytic mechanics around 1700. In fact, following the analysis by Pierre Duhem, del Monte has often been perceived in the literature as a pedant who worried over insignificant factors in the case of the equilibrium of the balance, for example, and posed a major stumbling block in the transition from statics to a science of motion by arguing that the relations valid in the case of equilibrium are not valid for motion. According to current interpretations, it was left to Galileo to lift that block in a bold move culminating with the 1632 *Dialogo* and especially the 1638 *Discorsi*. Several historians later in the century followed Duhem's views.[1]

However, there are many ways to look at Guidobaldo's contributions to mechanics and historically there have been many ways to look at *Mechanicorum liber*. My contribution does not aim at an exhaustive survey but rather explores four moments of the fortune—or, one could say, at times misfortune—of Guidobaldo's legacy in reverse chronological order. I start from a brief analysis of Pierre Duhem's views at the beginning of the twentieth century. I then move to the mathematician and also historian of mechanics Joseph-Louis Lagrange, whose historical introductions dating from ca. 1800 to the different editions of his *Mécanique analitique* contain valuable analyses of the development of mechanics and of Guidobaldo's contributions in particular. I continue my journey

[1] For the pervasive nature of Duhem's views see (Rose 1975, 233; Drake and Drabkin 1969, 46; Wallace 1984, 204–5, 241).

backwards in time moving to the seventeenth century, especially to the French mathematician Pierre Varignon, who was inspired by René Descartes to challenge the systematization of mechanics offered by Guidobaldo. Both Varignon and Descartes questioned whether it was acceptable to take the lever as the starting point of statics and sought instead more abstract and general principles. Lastly, I reach Galileo and his mentor Guidobaldo; I consider different aspects of their relationships, arguing that while in some respects Galileo broke with his mentor, in others he followed him quite closely.

I consider different ways of practicing mechanics: one, with which perhaps we are more familiar, relies on principles—increasingly more abstract and general—from which the solution to different problems can be derived; the other way relies either on the established example of the lever, or on other examples in different fields, and seeks to employ them to solve more complex problems by showing that they can be reduced to simpler cases. Instances involve showing that the winch or the inclined plane can be reduced to a lever, or that the motion of projectiles can be reduced to a special case of falling bodies. In conclusion, I wish to argue that, despite significant shortcomings, from both perspectives Guidobaldo played a more significant role in the development of seventeenth-century mechanics and the science of motion than has been generally acknowledged.

5.2 Duhem and the Punctilious Scholar

The French historian and philosopher of science Pierre Duhem has portrayed an influential image of Guidobaldo that has dominated the entire twentieth century. In his two volumes on *Les origines de la statique* (Paris, 1905-1906), Duhem provided a comprehensive account of the discipline across the centuries. In his account Duhem made of Guidobaldo a mediocre pedant or, in his words, a "narrow-minded" and "punctilious" mind eager to quibble over matters of little or no significance while disregarding valuable insights provided by the intuition of his medieval predecessors. Overall, Duhem was eager to promote the Middle Ages over the Renaissance: Guidobaldo's allegiance to the Greeks and dislike for medieval scholars such as Jordanus of Nemore did not fare well with the French historian. Moreover, in *Les origines de la statique*, Duhem was quite interested in results, whereas Guidobaldo showed greater sensitivity to the rigor and coherence of proofs and methods, or the foundational aspects of mechanics: in the case of the problem of the equilibrium of weights on the inclined plane, for example, del Monte preferred the problematic solution by Pappus of Alexandria—the Greek mathematician of the fourth century CE—over the more satisfactory result by Jordanus. I have deliberately used the term *result* by Jordanus, since his method has been considered problematic in that according to some he introduced in the proof the result he wished to demonstrate. Be that as it may, Jordanus did not rely on the lever in order to account for the inclined plane, as advocated by Pappus and del Monte, but rather sought an independent solution. I shall discuss del Monte's solution below. Since I have dealt elsewhere with Duhem's work and its pervasive influence, I can be rather brief here.[2]

In *Mechanicorum liber* Guidobaldo discussed at great length the problem of the equilibrium of the balance in which the center of suspension and the center of gravity coincide. One may question what is the general significance of this problem, given that it does not seem to be of central importance to the history of mechanics. In fact, the issue is quite subtle because it does involve an important methodological point concerning the problem of rigor and of approximations in the transition from mathematics to *physica* or the study of nature: in the sixteenth century it was not immediately clear which factors had to be included and which ones could be neglected, what was a suitable and acceptable approximation, and which approximation introduced significant errors in the result. Thus I would argue that although the problem of the equilibrium of the balance in the panorama of studies of sixteenth-century mechanics cannot be seen as crucial in terms of results, it did have broader methodological implications.[3]

At first del Monte's lengthy discussion seems paradoxical: in the opening of his treatise, he had argued that the key notion to study the equilibrium of the balance is that of center of gravity, which does not change by rotation. Therefore, if we rotate a balance suspended by its center of gravity, the equilibrium conditions are not altered and the balance remains stable in any position in which it is left, or is in a position of indifferent equilibrium. Later, however, del Monte challenged the opinions of his predecessors Niccolò Tartaglia, Gerolamo Cardano, and Jordanus of Nemore, who had argued that the balance returns to the horizontal position. In the course of his refutation of their views, del Monte seems to defend a different position, namely that the balance, far from returning to the horizontal position, tips over until it is perpendicular to the horizon. In justifying his reasoning del Monte introduced the notion that, strictly speaking, the lines of descent of the heavy weights of the balance are not parallel among themselves, but converge to the center of the earth. It is worth recalling that he was not the first to raise this issue of the convergence of the lines of descent: Tartaglia, for example, had mentioned it only to conclude that the amount of the deviation from the perpendicular was too small to be of any significance. Thus del Monte seems to contradict himself in arguing, first, that the balance is in a position of indiffer-

[2] See (Bertoloni Meli 2006, 26–30) and P. Duhem, *The Origins of Statics*, published originally in 1905–6, transl. in (Duhem 1991, 151–152).

[3] See (Bertoloni Meli 2006, 10–12, 32–35). On this topic see also (Gamba and Montebelli 1988, 213–250; Damerow, Renn, and Rieger 2001; Palmieri 2008, 302).

ent equilibrium, and then that it tips over. In fact, a more careful analysis of the text shows that Guidobaldo used the convergence of the lines of descent as part of an intellectual and rhetorical strategy. He did believe that in the case of a weight attached to one arm of a balance, that weight would descend not exactly perpendicularly, but at a tiny angle. In the case of two weights, however, he considered the center of gravity of the balance, which remained in the same position when the balance is rotated (see Figure 5.1).

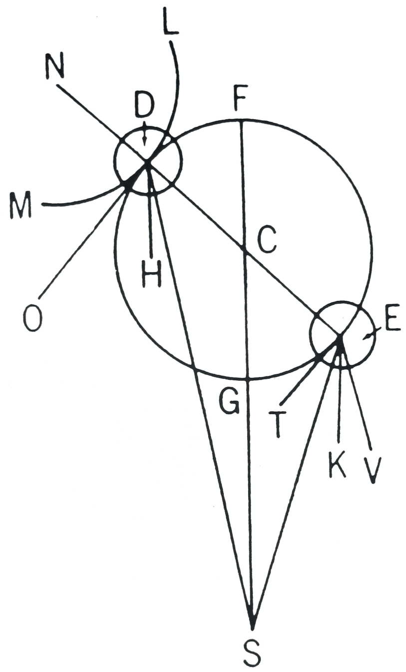

Figure 5.1: Del Monte and the convergence of the lines of gravity

We can gain a deeper understanding of del Monte's strategy by considering that Tartaglia and some of his contemporaries and predecessors had attached a physical role to the so-called "angle"—the so-called angle of contact—between the circle and its tangent, a magnitude that we now consider strictly nil but that was not considered so at the time, although it was considered smaller than any given angle. Thus it appears that Guidobaldo was taking into account and comparing different magnitudes in his approximations, implying that it is not legitimate to ignore the tiny but finite angle of convergence of the weights of the balance toward the center of the earth while taking into account the angle of contact, which in any case is smaller. But in the end, Guidobaldo's reasoning had a rhetorical stance, since he did not believe that that convergence played a role in the equilibrium of the balance anyway. In fact, Guidobaldo accepted the reasoning by Tartaglia, Cardano and their predecessors only as a concession in a typical Renaissance mode of argumentation, in order to show that even accepting their assumptions, their conclusions still would not follow.[4]

I wish to prevent a misunderstanding of my argument. Despite Duhem's problematic interpretation, it would be wrong to dismiss the issue of the direction of the lines of descent of heavy bodies as insignificant, since it did attract the interests of, and stimulated debates among, several mathematicians in Guidobaldo's time as well as in the seventeenth century and beyond. In his *Mechanica*, for example, John Wallis discussed the problem of the equilibrium of the balance and argued that if the line of suspension coincides with the center of gravity and the lines of descent converge to the center of the earth, the balance will be in stable equilibrium if it is parallel or perpendicular to the horizon, but will tip to the perpendicular position from any oblique position. Earlier in the century the problem was discussed in print and in correspondence among Guidobaldo, his contemporaries and immediate followers, as Enrico Gamba, Vico Montebelli, and more recently Sophie Roux have shown.[5] At the time of the French revolution, historian of mathematics Jean Etienne Montucla pointed out that if the lines of descent converge to the center of the earth, the center of gravity is no longer fixed but varies by rotation, for example, contrary to what Guidobaldo had thought; one may add that, in such circumstances, the very notion of center of gravity needs to be redefined.[6]

5.3 Lagrange and the Principles of Mechanics

In *Mécanique analitique* Joseph-Louis Lagrange included four historical sections on the principles of statics, hydrostatics, dynamics, and hydrodynamics. The emphasis on "principles" in the heading is revealing: in line with the approach to which he was a leading contributor in the second half of the eighteenth century, Lagrange conceived the history of mechanics as a history of principles and sophisticated mathematics. It is common lore among historians that Lagrange prided

[4] See (Bertoloni Meli 2006, 26–30). Van Dyck (2006) has independently reached similar conclusions. See also the contribution by Walter R. Laird in this volume. Paolo Palmieri has defended rather different views in (Palmieri 2008, 302).

[5] J. Wallis, *Mechanica*, in (Wallis 1693-9, vol. 1, 619 and 630–2). See also (Drake and Drabkin 1969, p. 47; Gamba and Montebelli 1988, 241–242; Roux 2004, 36–52).

[6] See (Montucla [1799]-1802, vol. 1, 691; Duhem 1991, 332–333).

himself in having written a treatise on mechanics without figures; this is indeed a significant feature from the standpoint of this paper, since he selected from del Monte's work an aspect related to his own perspective that appears of secondary significance in *Mechanicorum liber*, where the visual aspect was of central significance.[7]

Lagrange considered three principles of statics: the first is the lever, stating that the lever is in equilibrium if the attached weights are inversely to the distances at which they are hung; the second is the composition of motions, which we are going to discuss below in dealing with Varignon; and the third is virtual speeds, stating that the powers are in equilibrium when they are inversely as their virtual speeds, estimated in the same directions as the powers.[8] We may wonder what role Lagrange attributed to Guidobaldo in his scheme. In the historical introduction to statics in the 1788 *editio princeps* of *Mécanique analitique*, Lagrange simply ignored Guidobaldo. In later editions, however, he inserted two references to *Mechanicorum liber*. In the first, he argued that Guidobaldo was unable to apply the principle of the equilibrium of the lever to the inclined plane and the machines that depend on it: indeed, as we know from Duhem's criticism, this was a problematic area of *Mechanicorum liber*, one in which del Monte had followed the unsatisfactory approach of Pappus and that Galileo later sought to correct. With regard to the third principle, that of virtual speeds, Lagrange attributed a preliminary formulation of it to del Monte, when the Marquis stated that in equilibrium: "The space of the [moving] power is to the space of the weight, as the weight is to the power that supports it." ("Spatium enim potentiae [moventis] ad spatium ponderis eandem habet, quam pondus ad potentiam pondus sustinenes"). Guidobaldo's formulation may sound rather convoluted, but in fact the issue is quite straightforward if we start from the lever: the moving power or weight is to the moved weight inversely as the lengths of the arms of the balance and the distances covered are proportional to those lengths. The same applies to the other simple machines such as the pulley or the winch, since they can all be reduced to the lever, according to Guidobaldo. The merit of his presentation was precisely that it provided a general formulation: del Monte, however, stated his principle for individual simple machines, one by one. Despite its potential generality, he preferred to operate at a simpler level, as we are going to see.[9]

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[7] A critical analysis of Lagrange's historical work written by non-historians is (Capecchi and Drago 2005).

[8] See (Lagrange 1788, 8–11, at pp. 10–11). Lagrange provides a more sophisticated definition of this principle (Lagrange 1867-1882, vol. 11, 7, 19ff).

[9] Lagrange, *Mécanique* (1867-1882, vol. 11, 18–19); Lagrange referred both to *Le mecaniche* and to the *Discorsi* in the expanded 1656 edition (Lagrange 1811-1815, vol. 1, 7, 20; Monte 1577, ff. 43r–v with quoted passage, and f. 104v; Monte 1581, ff. 39r–v, 97r).

Lagrange's attribution has been fiercely questioned by Duhem; indeed in several instances Lagrange was rather swift in finding predecessors to this or that view. Recent scholars too, such as Edoardo Benvenuto, disagree with Lagrange. Benvenuto, however, found merit in Guidobaldo's principle linking the spaces described by the moving power and the moved weight and their respective power and weight, even though del Monte did not talk of virtual speeds and least of all of infinitesimal displacements. According to Lagrange, in his treatise on mechanics, *Le mecaniche*, first published by Marin Mersenne in 1634, Galileo extended Guidobaldo's individual statements and formulated a general principle for all simple machines, stating that the speed of the moving force is to the speed of the weight inversely as the weight is to the moving force: Galileo put together under one individual principle, stated in the opening of his work, what Guidobaldo had claimed for individual simple machines. Lagrange argued that John Wallis too adopted a version of this principle in his *Mechanica* of 1670–1.[10]

So far we have discussed Lagrange's views on the principles of statics. With regard to the science of motion or, as Lagrange called it, *la dynamique*, he attributed it entirely to the moderns, beginning with Galileo, excluding anyone before him.

5.4 Varignon, Descartes and the Rejection of Reduction

Let us move now to the remaining principle of statics according to Lagrange, that of composition of motion. In 1687—that fateful year—mathematician Pierre Varignon published a treatise addressed to the Paris Academy of Science, *Project d'une nouvelle mechanique*, in which he challenged del Monte's *Mechanicorum liber*. There are striking differences between Lagrange and Varignon: Lagrange discussed Guidobaldo from a strictly historical standpoint and paid special attention to his formulation of a general principle, an aspect that played a secondary role in *Mechanicorum liber* but that was of great significance to mechanics at the time when Lagrange wrote. Varignon treated Guidobaldo as a major figure in the field of mechanics, a proponent of an approach still worth considering, and, unlike Lagrange, examined Guidobaldo not for his formulation of a general principle of mechanics but rather for his practice based on the primacy of the lever. Varignon's publication raises several questions: In which sense was Varignon's project new? Why challenge a work first published in 1577, one hundred and ten years earlier, by an author who had died in 1607, eighty years before? Was Varignon's work related to broader concerns about the formulation and practice of mechanics at the time?

[10] See (Benvenuto 1991, vol. 1, *Statics and Resistance of Solids*, 80–1; Duhem 1991, 156–157; Galilei 1890–1909, vol. 2, 156–7, reprinted 1968; Galilei 1960, 148–149; Galilei 2002, 45–46).

A reader of Varignon's *Project* interested in new results, as opposed to methods, will be disappointed: Varignon's treatise is strictly methodological and foundational. In the preface Varignon states that he came across a letter in Descartes's correspondence in which Descartes argued that it was ridiculous to employ the principle of the lever to explain the pulley, as Guidobaldo del Monte had done in *Mechanicorum liber*.[11] The letter, now tentatively dated 1646, possibly addressed to the resident of the English monarch in The Hague, discusses various matters relating to mechanics and challenged Guidobaldo's attempt to reduce the pulley to the lever. It was in the brief treatise *Explication des engines*, however, that Descartes addressed the question of the foundations of mechanics understood as the science of simple machines in a more direct fashion. The *Explication* was appended by Descartes to a letter dated 5 October 1637 and addressed to Constantijn Huygens; the short treatise was first published in Paris in 1668 and then in Kiel in 1672. It is in that treatise that Descartes formulated the celebrated principle whereby the same force is required to raise a weight to a given height as to raise half that weight to double the height. Lagrange later considered Descartes's principle to be related to Galileo's principle expressed in *Le mecaniche*, first published by Marin Mersenne in 1634, and then in the 1656 edition of the *Discorsi*.[12]

In *Project d'une nouvelle mechanique* Varignon recognized that the Oxford Savilian professor of Geometry John Wallis had adopted a different approach, but this too he deemed conceptually not entirely satisfactory. In *Mechanica, sive de motu tractatus geometrico* of 1670–1, Wallis had followed an approach derived from Galileo and also from Guidobaldo—as pointed out by Lagrange. Wallis, however, did not share del Monte's primary concern and method of reduction of all simple machines to the lever.

Varignon argued that the lever has no privileged status over other simple machines; besides endorsing Descartes's rejection of the reduction of the pulley to the lever, he also questioned whether the lever had any link with the inclined plane, since both Guidobaldo and—in a more sophisticated fashion—Galileo had sought to reduce the problem of equilibrium of weights on an inclined plane to the lever or equivalently the balance, which is just a special example of a lever. His seems a curious statement given that in *The equilibrium of planes* Archimedes had provided an axiomatic theory of the equilibrium of the balance and therefore—at least historically—the lever did have a different status from that of the other simple machines. Thus Varignon was looking for a new and more abstract princi-

[11] "In trochlea autem ineptum mihi videtur vectem quaerere; quod si bene memini, Guidonis Ubaldi figmentum est" (Descartes 1897–1913, vol. 4, 696); see also (Duhem 1991, 421).

[12] See (Lagrange 1788, 9). If this were so, it would be extremely interesting in view of the fact that Galileo's principle was clearly inspired by Guidobaldo, as his *Le mecaniche* was inspired by *Mechanicorum liber*.

ple on which to base the science of mechanics of simple machines, one principle from which all simple machines could be explained and accounted for, without having to rely on one of them to explain the others. He found this principle in the composition of motions, stating the common parallelogram rule whereby the composition of two motions is directed along the diagonal of the parallelogram formed by the motions and has the length of that diagonal: the principle itself was not new, but the idea of using it in a foundational role was a novelty, as Lagrange was to point out in his 1788 historical account. Thus in this respect Varignon was justified in calling his work a "project for a new mechanics" (Lagrange 1788, 5–6). The best way to illustrate Varignon's method is to show some of his diagrams linking the *principle* of composition of motion to a study of simple machines (Figures 5.2 and 5.3).

As Descartes's and Wallis's cases show, Varignon was not isolated during the seventeenth century in seeking new and more general or abstract principles of mechanics. However, despite its inaccuracies, I believe that in this field Guidobaldo's *Mechanicorum liber* was still a notable source from a methodological standpoint. Guidobaldo had identified and addressed the problem of the foundations of mechanics: in order to be a *science*, mechanics could not be a heterogeneous collection of problems and *ad hoc* solutions, but had to be structured as a coherent body of knowledge descending from sound and widely accepted principles. For del Monte this anchor of certainty was to be found not in a new abstract principle but in the classical tradition and Archimedes's theory of the lever. By identifying levers in disguise—as we are going to see in the following section—in the simple machines, mechanics could expand to new and more challenging domains while at the same time retaining its certainty due to its link to Archimedes.

There is also another aspect worth considering at this point. First Galileo, and then others after him, had followed in part Guidobaldo's approach either in statics or—and this is a crucial point—in other domains of mechanics and the science of motion. As I have argued in *Thinking with Objects*, the practice of relying on some objects or cases to explain other more complex ones was quite widespread in the entire domain of the science of motion. It was this practice that was becoming less popular at the time of Varignon, because of the growing complexity and mathematization of mechanics on the one hand, and of the search for more general and comprehensive principles enabling mathematicians to go beyond the limited domains captured by the doctrine of the lever.

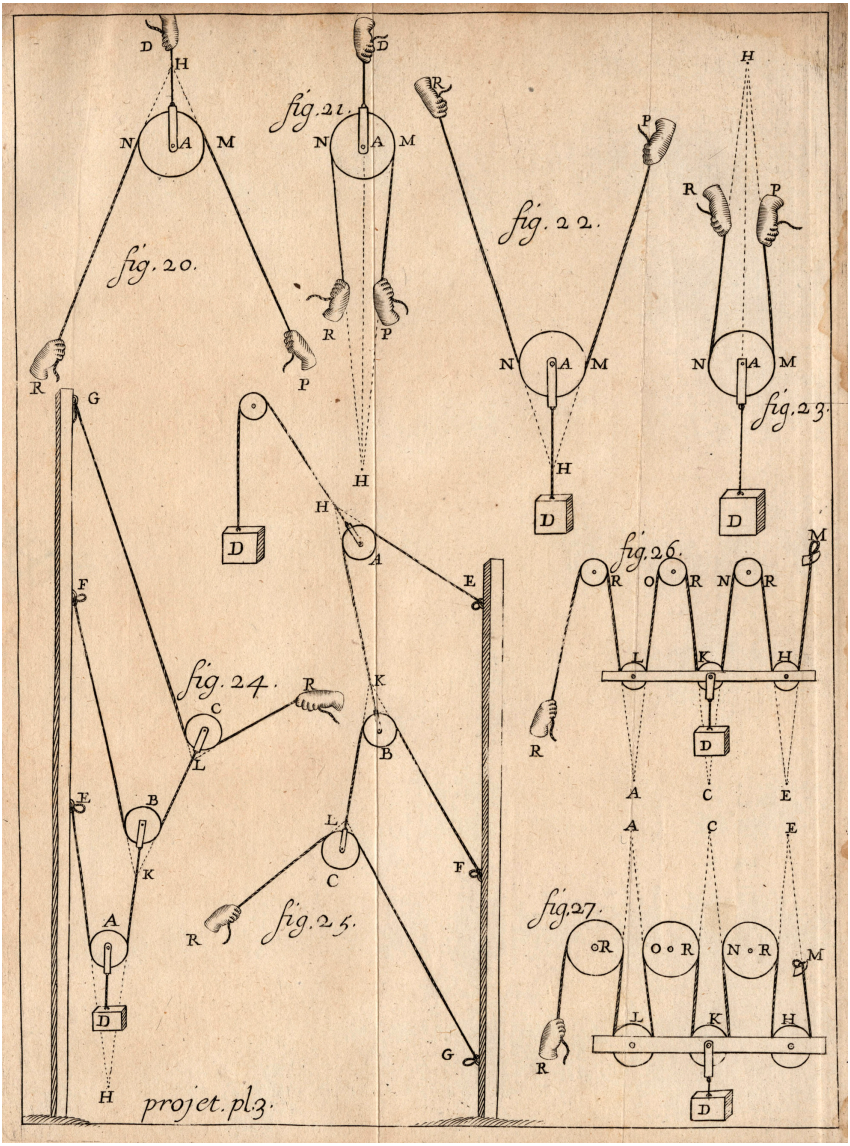

Figure 5.2: Varignon's principle

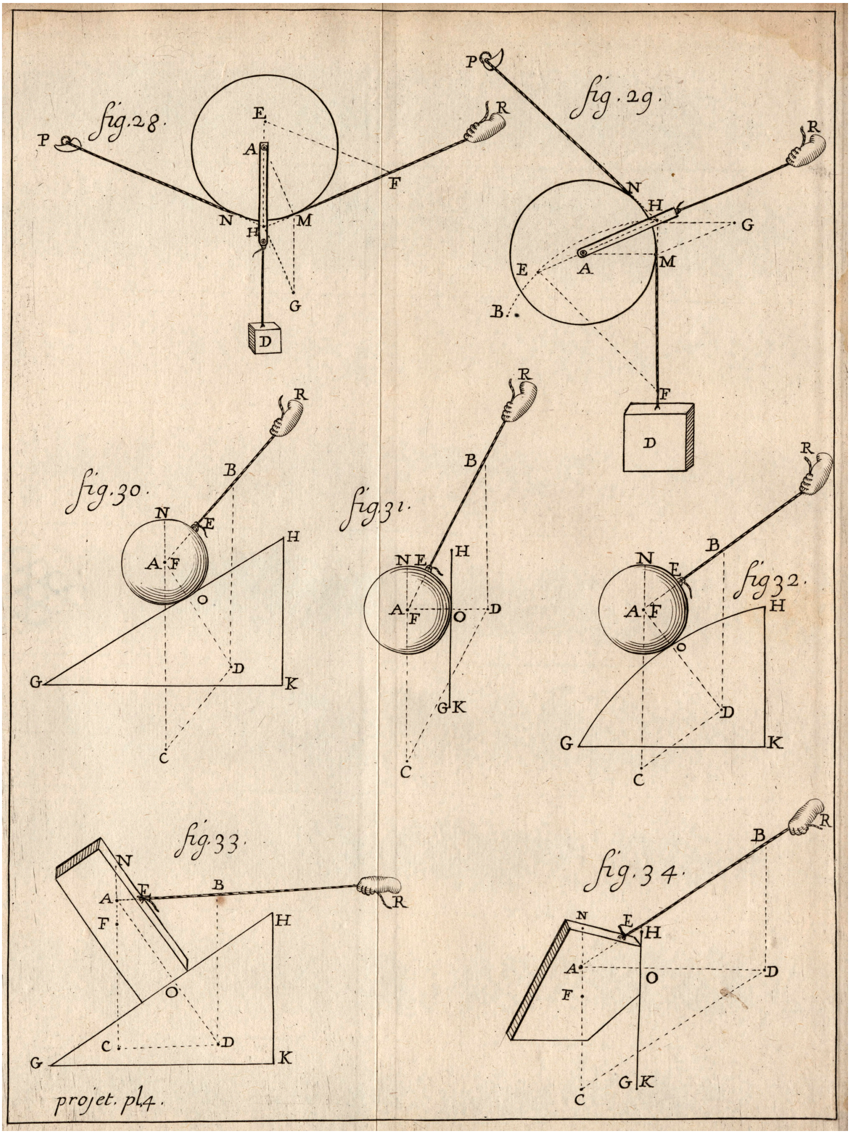

Figure 5.3: Varignon's principle

This departure from Guidobaldo's way of proceeding is exemplified by Newton's idea of explaining orbiting bodies in terms of projectiles (Figure 5.4). In a preliminary popular version of the third book of the *Principia mathematica*—symbolically published in the very same year of Varignon's treatise—Newton had included a diagram to this effect. This way of practicing mechanics and the science of motion, however, was perceived in a different way from the time of Guidobaldo. It is significant in this respect that Newton never published his diagram, which came to light only in 1728, one year after his death, when it was published not as a piece of current research but more like an addition to the shrine of Newtoniana. This famous diagram shows that orbiting bodies are projectiles by relating them directly in a visual way. In the text, however, Newton studied all cases starting from abstract principles—his famous three laws and collateral assumptions—in a style not dissimilar from that adopted by Varignon in his project for a new mechanics.

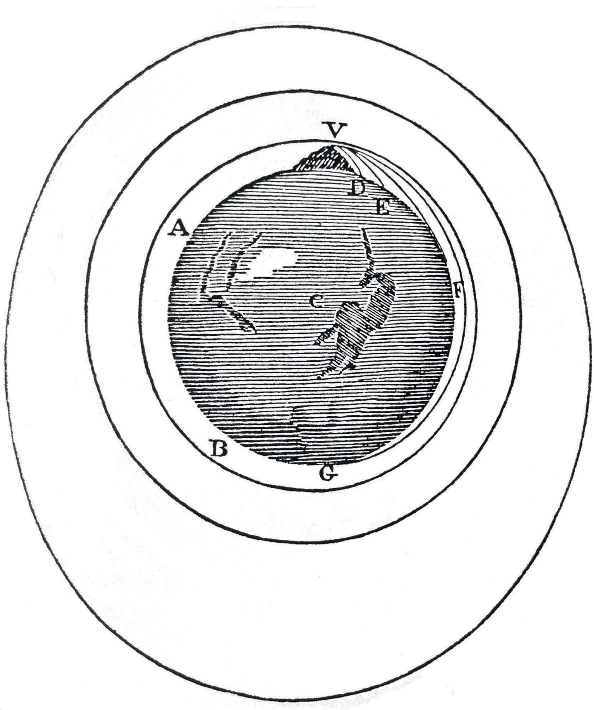

Figure 5.4: Newton's diagram

5.5 Guidobaldo, Galileo and the Practice of Mechanics

Important as principles are, I believe that they should not be seen as the only way of practicing mechanics, especially at a time when mathematicians often reasoned by analogy—whether this was rigorous or not is not my primary concern here—and explicitly advocated a way of doing mechanics based on the conceptual and practical manipulation of objects. While del Monte did attempt to formulate a principle of mechanics anchored to the objects or devices under investigation, he also invoked and applied in practice a visual hands-on approach that eschewed abstract principles in favor of concrete techniques and methods of proceeding. It is to his own work that I now turn.

Guidobaldo was the heir of a tradition, going back to Pappus and beyond, seeking to account for all simple machines in mechanics in terms of the lever. A similar approach can be found in other ancient texts in mechanics, such as the *Quaestiones mechanicae* then attributed to Aristotle and now considered to be the work of one of his early followers; that work, however, does not consider the simple machines and at times seems more concerned with the theme of wonder at the properties of the circle and balance than with rigorous proof. Moreover, *Quaestiones mechanicae* deal with a range of problems and not all of them can be reduced to the balance. It is in Book 8 of the *Collectiones mathematicae* by Pappus that one finds the most complete and influential treatment of mechanics based on the lever and it is no accident that Guidobaldo took Pappus as his master. The work had not been published yet but in all probability Guidobaldo had access to the translation by his teacher Federico Commandino that he was to see through the press in 1588, eleven years after *Mechanicorum liber* was published. First, one may ask, why start from the lever? The answer to this question was straightforward for both Pappus and Guidobaldo: the doctrine of the lever had been formulated and formalized by Archimedes in *On the equilibrium of planes* and was therefore the bedrock of mechanics: it was impossible to go beyond "divine" Archimedes in terms of authority and certainty. But it seems fair to argue that besides relying on historical precedence and authority, del Monte and others saw in the lever the archetypal mechanical device conceptually as well.

Another question one may ask is in what way the lever was used. There is no better way than to look at a concrete example, such as the winch: in this instance, del Monte started from an engineering diagram of the actual device shown in perspective. Then he showed a geometric section of the same and through this geometric diagram he showed visually in a process that I have called "visual unmasking" that lurking inside the winch one could detect a lever: this is what Guidobaldo called "reduction" of the winch to the lever (Figure 5.5). Another celebrated and in this case problematic example is that of the inclined plane. Here too Guidobaldo sought to find a lever in disguise, but in this case his solution ended up being problematic at many levels.[13]

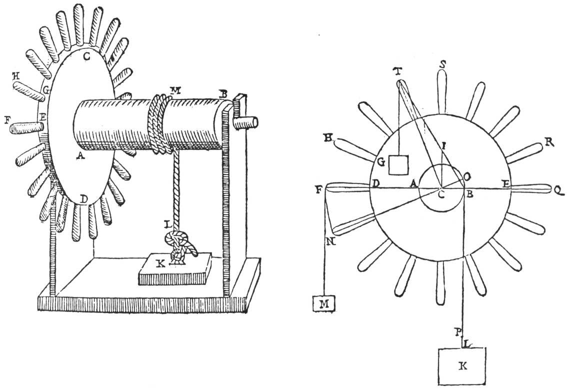

Figure 5.5: Guidobaldo on the winch

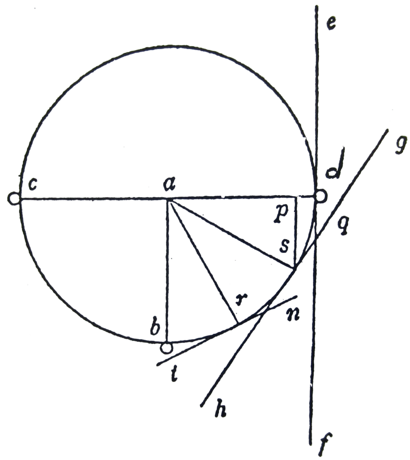

Figure 5.6: Galileo and the inclined plane

[13] See (Bertoloni Meli 2006, 24; Henninger-Voss 2000, 233–59, at 251–2).

Guidobaldo talked explicitly of a "reduction" of simple machines to the lever and the cases we have just seen illustrate his important concept. Looking at *Mechanicorum liber*, one does not get the impression that Guidobaldo worked from abstract principles, like those at the center of Lagrange's historical reconstruction, although he did formulate an abstract principle in terms of moving power, weight to be moved, and time. Rather, he worked from a form of visual reasoning in which the geometric diagram occupied center stage. Lagrange's emphasis on his rejection of figures highlights the gulf between these different ways of conceiving mechanics.[14]

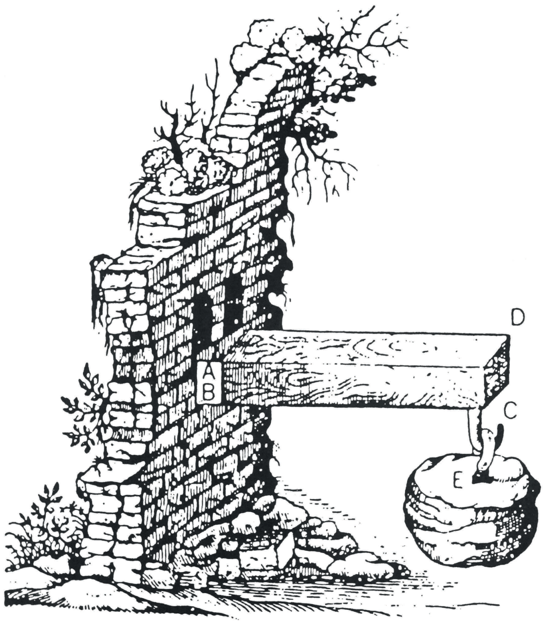

Figure 5.7: Galileo and the beam

Moving on to Galileo now, we notice a tension between Guidobaldo's program and Galileo's own work. I believe that initially Galileo sought and hoped to be able to fix the problems in Guidobaldo's account and then to extend the methods and ideas of his mentor to new domains, notably the science of the resistance of materials and the science of motion. We have seen above that del Monte's

[14] See (Monte 1577, f. 105v).

treatment of the inclined plane was defective. In providing a different viable account, Galileo attempted to rely on the same notion of "reducing" the inclined plane to the lever, though in this case the situation was slightly more complex in that Galileo had to draw an auxiliary balance (Figure 5.6) *cas* or *car* with bent arms, where the arm *as* is perpendicular to the inclined plane *hg*, and the arm *ar* to *tn*, rather than just uncovering or unmasking it; on the basis of simple geometry he could conclude that the weights of the bodies on the inclines are in equilibrium when they are inversely as the lengths of the inclines. As to the science of resistance of materials, Galileo's method is strikingly similar to Guidobaldo's: he sought to identify a lever lurking in the geometric diagram of a beam protruding from a wall (Figure 5.7), where B is the fulcrum and AB and BC are the arms.[15]

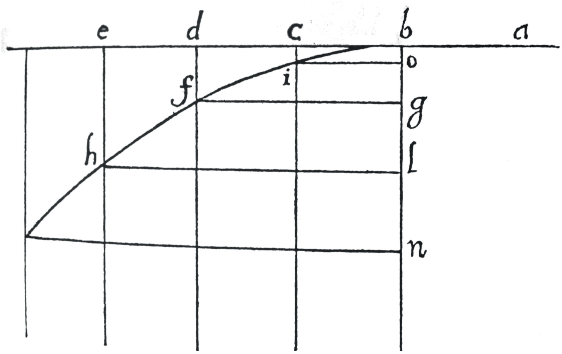

Figure 5.8: Galileo's parabolic trajectory

Galileo's early attempts to deal with the science of motion rely on the balance too, but Galileo soon realized that this path was not viable. He struggled for many years—even after the publication of the *Discorsi* in 1638, aged seventy-four—seeking a so-called mechanical foundation for the science of motion. Then in 1638 he put forward a new science relying on a definition and an axiom or postulate that were unrelated to the lever. Other portions of the science of motion, however, relied on visual techniques similar to those used by Guidobaldo. In the case of projectiles, Galileo showed lurking inside (Figure 5.8) a parabolic trajectory *bifh* a falling body, identified by the odd-number rule *bogln*: thus he was able to show that the violent motion of a projectile and the natural motion of

[15]See (Galilei 1890–1909, vol. 1, 298; Galilei 1960, 64–65; Bertoloni Meli 2006, 54–55 and 93). See also (Bertoloni Meli 2010a).

a falling body were just variants of each other, the difference being a horizontal projection.[16]

Similar techniques can be found throughout the century, in different forms. From Robert Hooke, who considered the orbital motion of a planet analogous to that of the bob of a conical pendulum, to Domenico Guglielmini, who saw a river as a more complicated version of a container filled with water with a hole at the bottom, this form of proceeding was pervasive. Although Guidobaldo was not the inventor of this style of work, he was certainly one of the most coherent proponents: it is through his work and in part Galileo's that we gain the best sense of this practice throughout the seventeenth century.

5.6 Concluding Comments

Guidobaldo was concerned with rigorous foundations of mechanics as opposed to a set of solutions to individual cases of this or that simple machine. In metaphorical terms, we can see Guidobaldo's program like a glacier slowly moving forward, solidly secured to its Archimedean sources represented by the theory of the lever. Guidobaldo saw the virtue of keeping the glacier intact even at the cost of limiting the areas he could explore, thus excluding a mathematical science of motion, for example.

In seeking to extend Guidobaldo's method to new areas while, at the same time, remaining committed to the primacy of the lever, Galileo realized that he had to make a choice: either remain committed to del Monte's strict program without being able to deal with new domains such as the science of motion, or extend his domain, breaking with the lever and del Monte's tradition. Galileo chose the latter, though he remained deeply committed to the problems of rigor and foundations in an Archimedean tradition. Thus with Galileo the glacier fractured and gave rise to a series of icebergs, isolated from the theory of the lever: within those icebergs, one can still detect a method similar to del Monte's, of explaining more complex problems by having recourse to simpler ones, often in the same visual fashion practiced by del Monte or in analogous ways. This method continued to be practiced in a wide set of domains for a large portion of the seventeenth century, and at times even beyond. Those floating icebergs were more and more isolated and distant from the glacier, but they were still viable areas within which to apply similar techniques. By the time of Newton and Varignon, many of those icebergs had become pools of cold water: mechanics was no longer practiced by analogy and visually, but rather by relying on increasingly abstract

[16] See (Galilei 1890–1909, vol. VIII, 198, 205–208; Galilei 1974, 154–165; Bertoloni Meli 2006, 50–60 and 66–104; Bertoloni Meli 2010b, 23–41). See also (Laird 1997).

principles with the usage of more and more sophisticated mathematical tools including infinitesimal geometry and the new analysis of differential equations.

Thus, although Lagrange may have overstated his case, Guidobaldo left a more lasting legacy to a larger portion of seventeenth-century mechanics than surmised by Duhem, both with regard to the reduction of complex objects or devices to simpler ones—whether this was a lever or not—and to the formulation of principles, not as abstract and general as Lagrange had believed, but solidly anchored to specific devices. Galileo shared the concern for rigorous foundations and played a key role in expanding and modifying his mentor's legacy with regard to both traditions: he relied on the process of unmasking simpler objects lurking inside more complex ones, and formulated some principles securely anchored to specific devices.

In conclusion, Lagrange and Varignon proved perceptive historians in identifying del Monte's concerns and practice, and in seeing him as a significant figure in the history of mechanics. Their works offer strikingly different perspectives from Duhem's and provide material for critical reflection and analysis on the changing horizons of mechanics.

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