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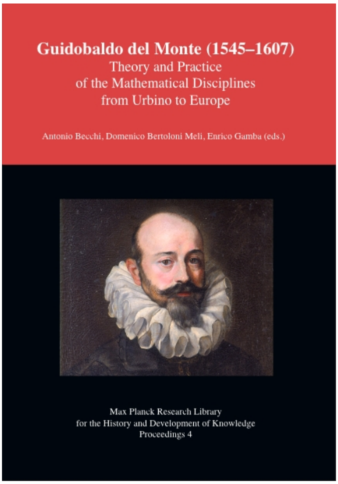
Kirsti Andersen:

Guidobaldo: The Father of the Mathematical Theory of Perspective



In: Antonio Becchi, Domenico Bertoloni Meli and Enrico Gamba (eds.): *Guidobaldo del Monte (1545–1607) : Theory and Practice of the Mathematical Disciplines from Urbino to Europe*
Online version at `http://edition-open-access.de/proceedings/4/`

ISBN 9783844242836
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Printed and distributed by:
Neopubli GmbH, Berlin
`http://www.epubli.de/shop/buch/27498`

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at `http://dnb.d-nb.de`

Chapter 7
Guidobaldo: The Father of the Mathematical Theory of Perspective

Kirsti Andersen

7.1 Introduction

When Guidobaldo Marchese del Monte is mentioned in the history of science it is most often for his contributions to mechanics. Thus, in the first edition of the *Dictionary of Scientific Biography* Paul Lawrence Rose devoted very few words to Guidobaldo's contributions to mathematics in general and to perspective in particular. Actually on the latter subject Rose stated simply that Guidobaldo had made "the best Renaissance study of perspective (1600)," leaving his readers in limbo about the contents (Rose 1974, 488). The *New Dictionary of Scientific Biography* contains a description of Guidobaldo's enrichment of perspective—put forward in his book *Perspectivae libri sex* (1600) (Andersen and Gamba 2008, 176–178). In the present chapter I elaborate on this description by discussing Guidobaldo's work on perspective in the context of mathematical approaches to perspective that are likely to have inspired him, especially focusing upon the use of convergence points for images of parallel lines before Guidobaldo's time. This is a natural subject to consider in connection with Guidobaldo's work because his major contribution was to provide an understanding of the geometry behind perspective, and within this setting the role of convergence points, later called vanishing points, became important.

My plan is to show that Guidobaldo deserves the title of "The father of the mathematical theory of perspective," as I have given him in this paper. I shall also make it clear that his insights did not come to him so easily and that he did not seem to have completely realized what a powerful instrument he had added to the toolbox for dealing with the theory of perspective.

7.2 Guidobaldo's Possible Sources of Inspiration

In surveying the literature on perspective with a geometrical approach that appeared before Guidobaldo's *Perspectivae libri sex* I only consider Italian works, because although authors North of the Alps published on perspective, none of them showed an interest in providing their readers with a mathematical understanding of the subject. The starting point is a work that does not have a particular mathematical approach, but is important because, directly or indirectly it is the source of inspiration for all other Italian literature on perspective. The work in question is Leon Battista Alberti's manuscript *De pictura* written in 1435. This work contains the first known definition of what we would call a perspective projection (Figure 7.1); Alberti himself did not use the word perspective but it became common to apply this word later in the fifteenth century (Andersen 2007).

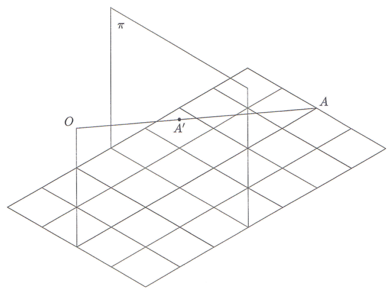

Figure 7.1: Alberti's way of introducing what soon after him was called perspective involved the optical concept of a visual pyramid and the concept of a section in a visual pyramid. In a simplified version it corresponds to saying that for a given point A, its image in a given picture plane π with respect to a given eye point O is the point A' where the line OA intersects π.

Alberti did not only describe a model for drawing in perspective; he also presented a correct construction of the perspective image of a grid of squares where one set of parallel lines is orthogonal to the picture plane—such lines I call *orthogonals* (Figure 7.2).

In his book he did not explain the geometry that led him to his construction claiming that he wanted to outline "as a painter speaking to painters [...] the first rudiments of the art of painting."[1]

[1] Alberti himself, however, seems to have wondered about this correctness, because he wrote that he had involved some geometrical explanations when presenting his construction to some of his friends (Alberti 1972, §23).

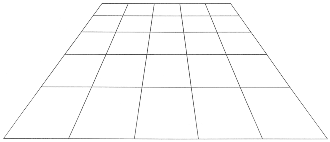

Figure 7.2: In *De pictura* Alberti describes how to make the perspective image of a grid of horizontal squares. This served as a kind of coordinate system that helped painters to decide where images of objects placed on a pavement with square tiles should be depicted in the picture plane.

There is no obvious way of reconstructing Alberti's geometrical arguments; however I am certain that they involved a rule which he took for granted and which is of interest for tracing Guidobaldo's sources (Andersen 2007, 23). This rule was that the images of *orthogonals* lines meet in one point (Figure 7.3).

Actually, Alberti also characterized the meeting point, namely as the orthogonal projection of the eye point upon the picture plane—which he called the *centric point* and which later was called the *principal vanishing point* (Figure 7.4).

The assumption that the images of *orthogonals* meet, if extended, in one point, I hereafter called the *convergence rule for orthogonals*—and I use the expression that the *orthogonals* converge at the point of intersection. This rule was actually applied without question by most pre-seventeenth-century authors on perspective and was one of the foundations of the majority of the various methods used for constructing the image of a horizontal square suggested by these authors.[2] Only the two Italian mathematicians, Federico Commandino and Giovanni Battista Benedetti avoided applying the convergence rule for orthogonals. The rule itself is an easy corollary from a theorem which Guidobaldo published in 1600 and to which I return in section 3.

Most of Alberti's successors writing on perspective followed his approach and avoided mathematical arguments, but a few authors were so engaged in mathematics that they attempted to provide their readers with a feeling of the geometrical aspects of perspective. Before Guidobaldo, these authors were all Italian,

[2]All authors before Simon Stevin (whose work on perspective appeared in 1605, see Stevin 1605a, 1605b and 1605c; Sinisgalli 1978, 167–344) introduced their construction methods as constructions of the perspective images of polygons—in most cases of squares—situated in a given horizontal reference plane. However, the constructions could also be used to construct the image of a given point in the reference plane.

the first being the painter and mathematician Piero della Francesca, whose book on perspective *De prospectiva pingendi*, was composed some time before 1482 (Piero 1942, 46).

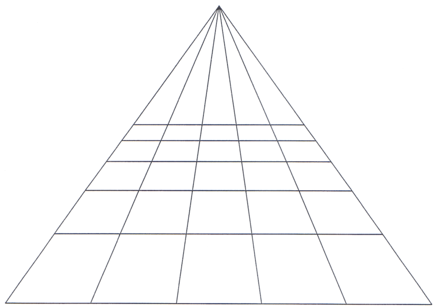

Figure 7.3: Illustration of Alberti's tacit rule: the images of orthogonals converge at one point in the picture plane.

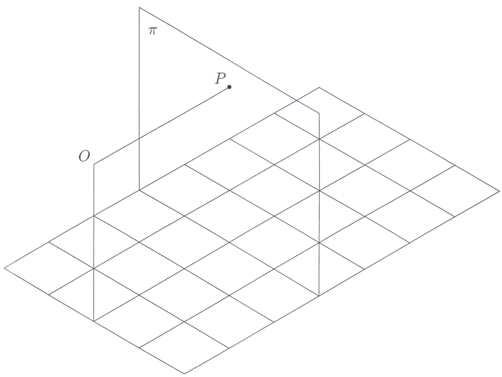

Figure 7.4: The orthogonal projection of the eye point O upon the picture plane, the point P, is Alberti's convergence point for *orthogonals*.

Besides taking the convergence rule for orthogonals for granted, Piero applied a rule stating that the images of a set of parallel horizontal lines forming an angle of 45° with the *orthogonals* have a convergence point (Figure 7.5).

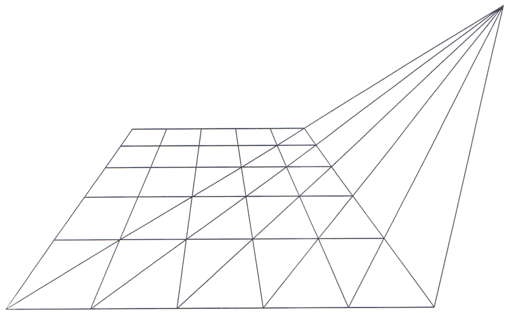

Figure 7.5: Illustration of the rule applied by Piero della Francesca: the images of diagonals are drawn so they converge in a point—later called a distance point. This point lies on the horizontal line which became know as the horizon and which is the horizontal line through the convergence point for *orthogonals*.

He also described the position of this—to which I return in connection with Vignola. Moreover, he attempted to prove this convergence rule but his proof was not convincing. That Piero could not come up with a satisfactory proof is not surprising, because he did not have the mathematical education necessary for doing so. The proof had to wait until Guidobaldo had created a mathematical foundation for perspective.

Piero's mathematical approach was followed by one architect and three mathematicians: the architect was Giacomo Barozzi da Vignola, whose work on perspective was edited by the mathematician Egnazio Danti, and the two other mathematicians were the already mentioned Commandino and Benedetti. These writers are all likely to have inspired Guidobaldo, but in different ways. Commandino's role was primarily to arouse Guidobaldo's interest in perspective, while Vignola's, Danti's and Benedetti's writings on perspective probably made Guidobaldo aware of other points of convergence in perspective compositions than the convergence point for the images of *orthogonals*.

Commandino's motivation to take up perspective was rather special since it was a spin-off of his main scientific occupation which was to translate and comment upon the classical Greek works on science and mathematics. Among the books Commandino edited was Ptolemy's *Planisphaerium*, whose theme was a certain central projection which later became known as the stereographic projection. Ptolemy applied this projection for mapping points on the celestial sphere upon the plane of the equator. While working with the projection, Commandino remarked that it is similar to a perspective projection and then devoted the first nineteen folios of his comments to perspective (Commandino 1558, fol. 2^r–19^r).

Although Commandino's presentation of the subject is very scholarly and includes exact proofs for all his mathematical statements, it does not resemble any other presentation of perspective and some parts of it are rather opaque, not to say downright awkward (Andersen 2007, 141–145). En passant, I would like to add that although Commandino stressed the conceptual relation between a stereographic and a perspective projection, there is no similarity between his treatments of the two projections. It may be that when he started working on perspective, he had hoped to gain more insight into stereographic projections, or vice versa that his knowledge on stereographic projection would have thrown more light on the mathematics behind perspective, but this did not happen. The fact that Commandino's treatment of perspective was far from intuitive may have contributed to some of the difficulties Guidobaldo had when he started working on perspective—as we shall see in section 3.

Danti's edition of Vignola's book on perspective appeared as *Le due regole della prospettiva pratica* (*The two methods in practical perspective*) in (Vignola 1583)—ten years after Vignola's death. For tracing ideas which may have inspired Guidobaldo, one of the two *regole* or methods referred to in this edition is of particular interest; the one that later was called a distance point construction. This method is based on the assumption, mentioned in connection with Piero della Francesca, that the images parallel horizontal lines forming an angle of 45° with the *orthogonals* converge at one point. To have a short name for the mentioned lines I call them *diagonals*—which was actually also done by Danti as we shall see shortly. There are two sets of *diagonals*, one verging to the right and one to the left—seen from the eye point. Vignola took for granted that the convergence points for the diagonals lie on the line known as the *horizon* which is the horizontal line through the principal vanishing point. Moreover, perhaps inspired by Piero, he assumed that the two convergence points were situated at a distance from the principal vanishing point that is equal to the distance between the latter point and the eye point. As this distance in general was called just *the distance*, the two convergence points came to be known as *distance points*. In Figure 7.5 is shown the right distance point.

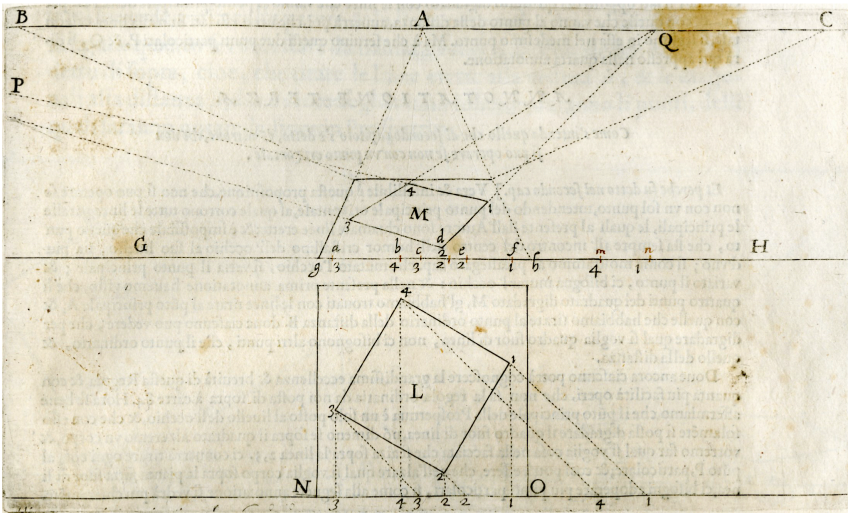

Figure 7.6: Vignola illustrating that other lines than orthogonals and diagonals can have convergence points (Vignola 1583, 115; with kind permission of the Biblioteca Oliveriana, Pesaro).

Vignola's application of the convergence rule for the *diagonals* seems to have inspired him to conclude that lines other than *orthogonals* and *diagonals* can have convergence points. Thus, in one of his examples (Figure 7.6) Vignola threw the square *L,* with no sides parallel to the horizon, into perspective and he let the images of both pairs of parallel sides converge on the horizon (although the sides *1,4* and *2,3* are not prolonged to the horizon, their point of intersection does lie on it). He included another interesting diagram (Figure 7.7) in which he drew diagonals in vertical squares so that they have convergence points (the points *C* and *E* in the figure). In his text Vignola did not mention the various points of convergence; in fact he wrote very few comments to his illustrations, apparently being of the opinion that it was easier to learn how to make perspective constructions by studying his drawings carefully than by reading a text.

His commentator, the mathematician Danti, started to wonder about converging points and included in a long introduction to Vignola's text the following interesting definitions involving the earlier-mentioned introduction of the term diagonal (definition VII).

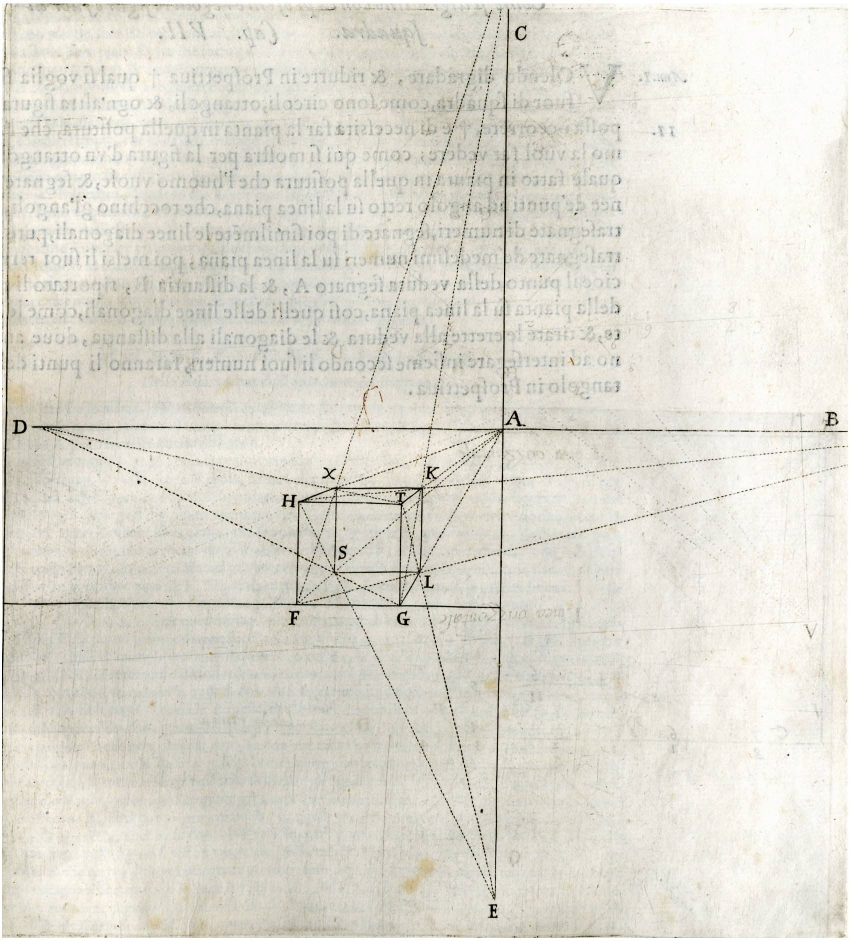

Figure 7.7: Vignola's illustration for his construction of a cube in perspective. It reveals how he applied the distance points *D* and *B* (though he did not include the latter point itself in his diagram) when he constructed the image of the cube's bottom square, and how he let the images of the diagonals in the two vertical squares that are not parallel to the picture plane converge in the points *E* and *C* (again, he did not include the latter point) (Vignola 1583, 107; with kind permission of the Biblioteca Oliveriana, Pesaro).

- Definition V
  Perspective parallel lines are those that will meet at the horizontal point [a point on the horizon].
- Definition VII
  A distance point is that at which all the diagonals arrive.
- Definition X
  Principal parallel lines are those which will all converge at the principal point of perspective [principal vanishing point].
- Definition XI
  Secondary parallel lines are those which will be united at the horizontal line at their particular points apart from the principal point.[3]

Thus, Danti assumed that not only the images of *diagonals* and *orthogonals* but the images of any set of parallel horizontal lines (apart from those parallel to the picture plane) converge at a point on the horizon. When formulating his definitions Danti apparently only thought of horizontal lines as converging. However, in his comments to Vignola's drawing reproduced in Figure 7.7, Danti acknowledged that the points C and E were converging points. With Danti's definition XI in mind, one gets rather surprised by his illustration reproduced in Figure 7.8.

In this he let one pair of the images of the parallel sides of the horizontal square P converge at the point G on the horizon whereas the two other sides do not converge at the direction of the horizon AG. He may, of course, have made a drawing error, but if so the diagram shows that it was not entirely clear to him that the images of parallel horizontal lines not parallel to the picture plane should meet on the horizon.

Benedetti described his thoughts about perspective in the treatise *De rationibus operationum perspectivae* (*On the reasons for the operations in perspective*) published in (Benedetti 1585). In this he proved the correctness of a couple of perspective constructions, one of which was later also presented by Guidobaldo. Moreover, he showed some understanding of general convergence points, but similarly to what Danti did in his diagram reproduced in Figure 7.8, Benedetti drew one pair of sides in the image of a horizontal rectangle having no sides parallel to the picture plane as lines converging on the horizon, whereas the second pair did not have a convergence point on the horizon (Andersen 2007, 149–151).

[3] Definitione quinta: Linee parallele prospettive sono quelle, ché si vanno a congiugnere nel punto orizontale. Definitione settima: Punto della distanza è quello, dove arrivano tutte le linee diagonali. Definitione decima: Linee parallele principali son[o] quelle, che vanno a concorrere tutte insieme nel punto principale della Prospettiva. Definitione XI: Linee parallele secondarie sono quelle, che vanno ad unirsi fuor del punto principale nella linea orizontale, alli loro punti particolari (Vignola 1583, 4–5).

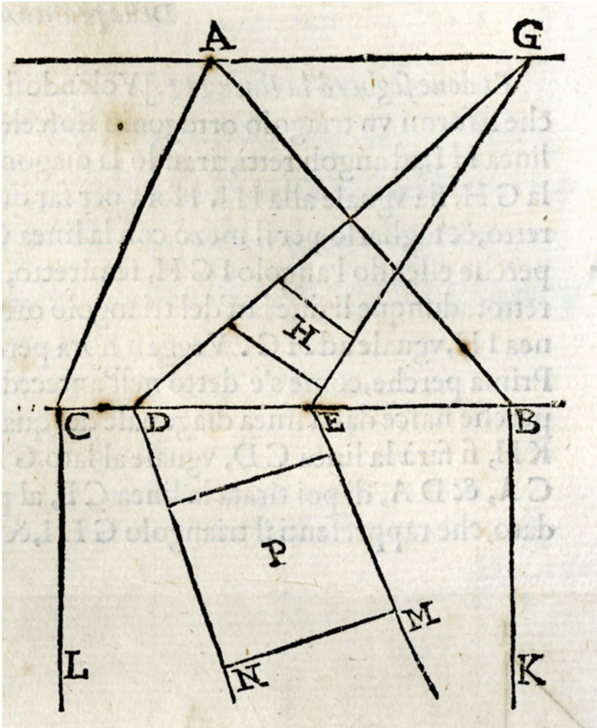

Figure 7.8: Danti's drawing of the perspective image *H* of the square *P* (Vignola 1583, 5; with kind permission of the Biblioteca Oliveriana, Pesaro).

Summing up the Italian developments before Guidobaldo, it can be concluded that some convergence points were applied—but there was no mathematical understanding of their property.

7.3 Guidobaldo Changing the Foundation of Perspective

Guidobaldo was a man of independent means and spent much time studying at his estate in Montebaroccio near Urbino. Here he had contact with Commandino who undoubtedly awakened Guidobaldo's interest in perspective. Thus, an unpublished manuscript on perspective which Guidobaldo presumably composed about 1588 reflects many of Commandino's ideas. Guidobaldo took up perspective a bit later in another manuscript and there he presented new ideas that resulted in his innovative approach to perspective. The information about the two manuscripts comes from Paola Marchi's thorough examination of them (Marchi 1998, 20, 72, 74–81). As indicated in the introduction, Guidobaldo had some problems in maturing his ideas, a fact he himself described in a letter he sent to Galileo in January 1593:

> My perspective is half asleep and half awake, because—to tell you the truth—I have so many engagements that I can scarcely breathe. For these matters I need to be free of all concerns. Still, I do want to finish it [...]. However, I have not yet discovered everything [...].[4]

By September, Guidobaldo thought it would be possible to finish his work during the winter and to have it published within a year (Galilei 1929-1939, vol. 10, 62). However, it was 1600 when the printed version of his work appeared—with the earlier mentioned rather down-to-earth title *Perspectivae libri sex*, but with a playful text written on a ribbon on the title page (Figure 7.9).

Besides being busy with various obligations, it seems that Guidobaldo spent much time trying to understand what made perspective work as it did, or in other words investigating the foundation of the theory of perspective. We can even, to some extent, follow his path to the great insight in his printed work, because there he included some of his early results despite the fact that they had been made superfluous by his subsequent research. An illustrative example of this is his treatment of a general convergence rule for parallel line segments.

Guidobaldo started by looking at parallel horizontal line segments like *BC*, *DE*, and *FG* in Figure 7.10 in which *A* is the eye point and *HKML* the picture plane—which is not parallel to the parallel line segments.

From reading Vignola, Danti, and Benedetti, Guidobaldo presumably had the idea that the images of *BC*, *DE*, and *FG*, that is *ML*, and *ON*, *QP*, when extended, converge at a point which he called *X* in figure. He wanted to prove this result, but at first he did not come up with a straightforward proof. His way out was to introduce an auxiliary picture plane *HRIL* parallel to the line segments *BC*, *DE*, and *FG* because that enabled him to apply a result he had proved earlier. Thus, he knew that in the picture plane *HRIL* the three parallel line segment would be depicted as parallel. From this result he concluded by a proof by contradiction that the images *ML*, *ON*, *QP* in the original picture plane *HKML* converge.[5]

It is worth noting that in this first treatment of parallel line segments Guidobaldo related the convergence point *X*, which he called *punctum concur-*

[4] "La mia Prospettiva mezzo dorme e mezzo vegghia, ché, a dir il vero, io ho tante le occupationi, che non mi lasciano respirare; e per queste cose bisognarebbe esser libero da ogni fastidio: pur la voglio finir [...]; ma non ho ancor trovato ogni cosa [...]" (Galilei 1929-1939, vol. XX, 54).

[5] For details see (Andersen 2007, 247; Marchi 1998, 94–103).

Figure 7.9: The title page of Guidobaldo's epoch-making book on perspective. The sentence in the ribbon I interpret as "without deception we are deceived."

sus, to the images of the parallel lines and not to the lines themselves. This way of conceiving convergence points he shared with his predecessors, and it has very appropriately been described by Marchi as considering a convergence point as an *operative point* in the construction of the images of the parallel line segments (Marchi 1998, 23, 37). I will now describe how Guidobaldo, later in his research, introduced this point in a different way.

In his subsequent investigations Guidobaldo extrapolated from his result concerning the images of horizontal parallel line segments and in this process he seemed to get more and more insight ending up with a completely general result concerning the images of any set of parallel line segments that are not parallel to the picture plane, for which he gave a very elegant proof (Andersen 2007, 249). However, in formulating his theorem he tended to keep to the wording that he had used in his first formulation of his convergence result. Thus, his proof for the general result is much more general than his formulation of it. In fact, Guidobaldo's proof establishes what I have called *the main theorem of perspective*.

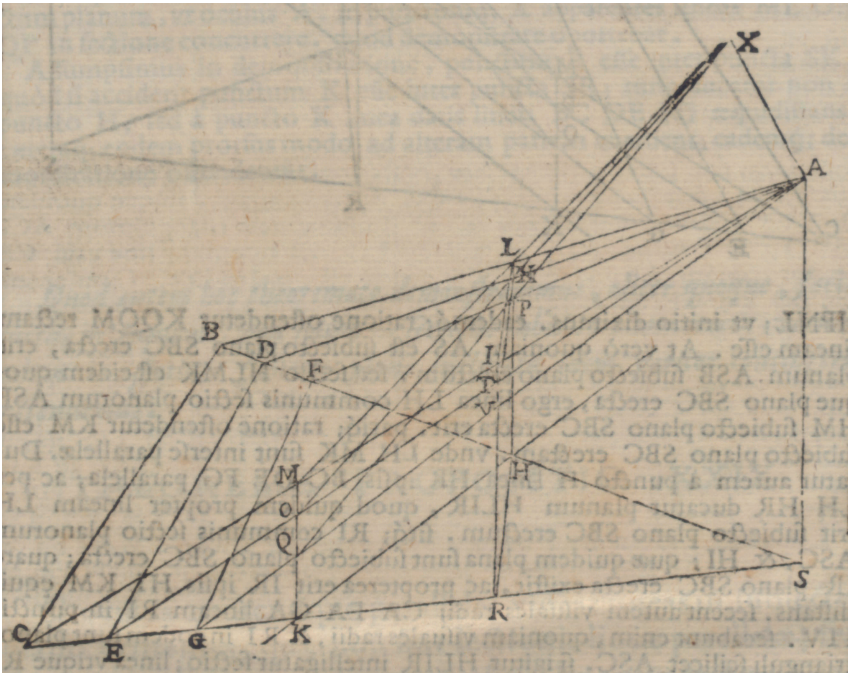

Figure 7.10: Guidobaldo's illustration for one of his proofs concerning a general vanishing point (Monte 1600, 35).

Before describing the content of this theorem, I would like to return to my discussion of the introduction of a convergence point. As we saw, Guidobaldo first characterized it as the point in which the images of parallel lines—not parallel to the picture plane—converge, and gave it the name convergence point. However, in his last proof for the general theorem he introduced the convergence point differently. First, he considered a situation in which there is given a picture plane π, an eye point O, and a line AB cutting the picture plane in A (Figure 7.11 the lettering is mine). Then he introduced the point V as the point of intersection of the plane π and the line through the eye point O parallel to the line AB (Monte 1600, 43; cf. Andersen 2007, 248). This point is, as we soon shall see, the same as the earlier convergence point, but now assigned to the line AB instead of to its image. In the beginning of the eighteenth century the British mathematician Brook Taylor assigned the point V to a line AB in the same way and called it the vanishing point of the line (Taylor 1715, 3)—a name that is still in use in English.

How can we prove that this vanishing point is the same as the previously considered convergence point? The answer is: with the help of the main theorem. It is hence the right place to present this theorem. In fact, the main theorem is very easy to formulate, as it states that the image of a line intersecting the picture plane like *AB* in Figure 7.11 is determined by its point of intersection *A* and its vanishing point *V*. Perhaps, this result does not seem very impressive, but it turned out to be almost revolutionary in the mathematical theory of perspective.

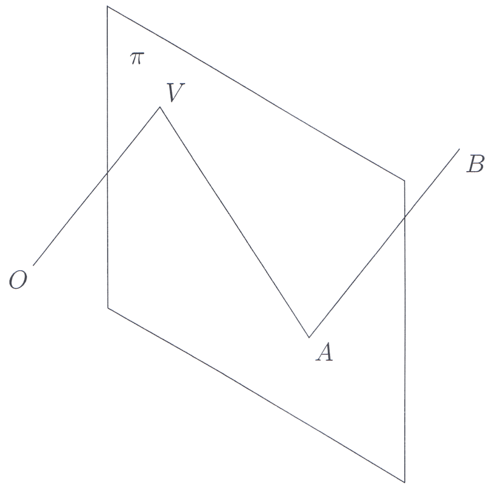

Figure 7.11: Diagram to illustrate Guidobaldo's general determination of the image of the line *AB*.

It is not immediately clear how we get from the result that the image of the line *AB* lies on the line *AV* to the relation between a convergence point and a vanishing point. However, if we consider a second line segment *CD* parallel to *AB* (Figure 7.12), we can conclude from the main theorem that this line segment has an image that is determined by the point *C* and the point in which the line through the eye point *O* parallel to *CD* cuts the picture plane π. Since *AB* and *CD* are parallel, the latter line is also *OV*, thus the image of *CD* lies on the line *CV*, implying that the images of the two parallel lines *AB* and *CD* meet or converge at the point

V. And for any other line parallel to *AB* and *CD*, the same argument shows that its image also passes through *V*. Hence the point *V* is indeed a convergence point.

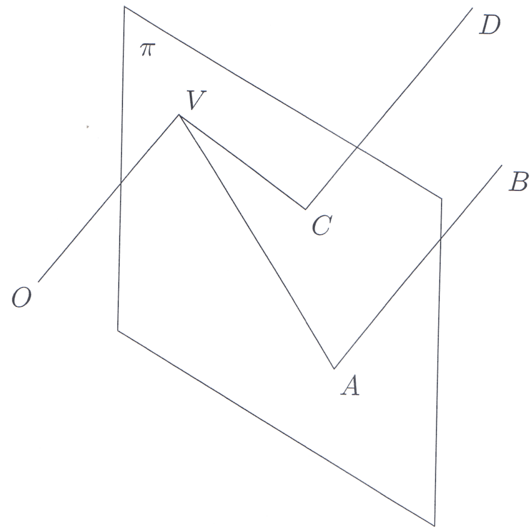

Figure 7.12: Diagram to illustrate that the two parallel lines *AB* and *CD* have the common vanishing point *V*.

I mentioned that Guidobaldo's proof of *the main theorem* was very elegant; it was also short and did not need any greater geometrical insight than the one he had inherited from classical Greek mathematics. Thus, it was not Guidobaldo's proof that gave rise to his new insight; rather it was the process of investigation that made him focus on convergence points and that led him to the concept of a general vanishing point—although he kept the name convergence point. This was really a crucial step in the history of the mathematical theory of perspective. With some simplification, it can be claimed that all further developments of this theory were a generalization of Guidobaldo's idea of a vanishing point for any set of parallel lines that are not parallel to the picture plane and of the *main theorem of perspective.*

As I noted in the introduction, Guidobaldo's result was more fundamental and had greater potential than he himself perhaps realized. And he was not the only one not to see how fundamental an insight he had reached. In fact, although *the main theorem* also constituted the primary theoretical tool for his successors, it took more than a hundred years before this was made explicit. The person to do so was Taylor, who in 1719 claimed "This Theorem being the principal Foundation of all the Practice of Perspective" (Taylor 1719, 14).

7.4 Guidobaldo's Main theorem and Perspective Constructions

Guidobaldo's new theory made it easy to conclude that Alberti and his successors were right when they constructed the images of *orthogonals* so that they pass through the orthogonal projection of the eye point upon the picture plane: According to Guidobaldo's definition of a vanishing point, the mentioned point is indeed the vanishing point of the *orthogonals*—the principal vanishing point. Similarly it can be concluded from the *main theorem* that the images of *diagonals* have one of the distance points as vanishing point.

To be able to make perspective constructions of objects it is sufficient to be able to construct the images of given points. However, all Guidobaldo's predecessors had presented construction of polygons, and Guidobaldo kept to this practice. Still, the methods he created were in principle point-wise constructions. Thus, from the image of a line, Guidobaldo turned to determining the image of a given point; he did this by constructing the images of two lines passing through the given point. He was so greatly taken by the possibility his *main theorem* offered him in choosing a pair of lines through a given point that he presented no less than twenty-three different methods of constructing the image of a point (Monte 1600, 61–104).

7.5 Guidobaldo's Other Contributions to Perspective

Apart from solving the problem of understanding the geometry behind perspective constructions, Guidobaldo touched upon a number of new themes in his *Perspectivae libri sex*; I briefly describe the three I find the most interesting, namely:

- How to perform perspective constructions in other picture planes than vertical ones, such as oblique and curved planes.
- How can we from perspective compositions gain knowledge about the original configuration?
- A subject I call *direct constructions* in the picture plane, whereby I mean constructions that are performed directly in the picture plane without involving auxiliary drawings such as a plan.

In giving up the requirement that a picture plane should be vertical, Guidobaldo started by explaining how to construct images in an oblique picture plane (Monte 1600, 136–151). His treatment of this subject was based on the *main theorem* which he had formulated so generally that it also includes oblique planes. He continued by studying constructions in rather spectacular screens which were composed of more surfaces, some of which could be curved. In one of his examples he went so far as to consider a picture plane formed by a surface combined of a cylinder, a sphere and a cone (Monte 1600, 164). Working with curved surfaces can become very complicated, but Guidobaldo kept to fairly simple examples.

The theme of getting information from a perspective drawing has become known as *inverse problems of perspective*. In general such problems have more mathematical than practical relevance since a perspective projection is chosen when one wants to give a visual impression of how an object looks. If one wants to convey a precise information about the shape of an object, another representation than a perspectival projection is chosen, for instance a parallel projection or a plan and an elevation.[6] There is, however one inverse problem of perspective that does have some practical interest, namely the following. Where should a person standing in front of a perspective image place his or her eye to properly perceive the scene created by the artist?

From a mathematical point of view, answering this question means reconstructing the eye point that the artist used for making his perspective drawing. The general problem of finding the eye point for a perspective composition is indeterminate, unless some information is given or some assumptions made. This may typically involve assuming that a tiled floor is the image of a floor with square tiles. However, rather than being inspired by practical questions or driven by a wish to gain mathematical insights, Guidobaldo seems to have taken up inverse problems because he wanted to use their solutions as auxiliary results for perspective constructions (Andersen 2007, 261).

Though Guidobaldo himself did not do much about inverse problems of perspective, he may have inspired several of the leading mathematicians in the field of geometrical perspective, including Stevin, Taylor and Johann Heinrich Lambert, to take an interest in these problems. Like Guidobaldo, they did not primarily treat the practical aspect of inverse problems; instead, they focussed upon characterizing the information needed to be able to solve a problem of inverse perspective uniquely. I have just given an example in which it was assumed that some tiles appearing in a picture were images of squares, the mentioned protagonists wanted to assume less.

[6] In the mid-nineteenth century the theory of inverse perspective underwent a revival and was further developed in connection with photographic surveying.

With respect to *direct constructions* Guidobaldo treated two examples in each of which he showed how one of the basic Euclidean constructions can be performed directly in the picture plane (Monte 1600, 113–114). In the first he assumed that in the picture plane were given the images of a line and a point in a horizontal plane. He then wanted to perform a direct construction of a line through the given point that is the image of the line that passes through the original of the given point and is parallel to the original of the given line. In his second example Guidobaldo treated the perspective version of the following Euclidean construction: Through a given point on a given line, draw a line that makes a given angle with the given line. The theme of direct constructions was taken up by the later perspectivists who developed Guidobaldo's idea and systematically treated the perspectival versions of the most common Euclidean constructions.

7.6 Guidobaldo's Influence on the Academic Approach to Perspective

Although Guidobaldo's accomplishment in the theory of perspective really deserves to be known, he himself seems to have been unable to pass on the message that his theory was helpful for those who wanted to understand the operations performed in perspective constructions. In September 1593 he wrote to Galileo that he was cutting and abbreviating his manuscript as much as he could, because he found it too long (Galilei 1929-1939, vol. 10, 62). However, Guidobaldo seems to have been so attached to his material that he could not restrict himself to the really important issues. The result was that his brilliant ideas drowned in a sea of irrelevant propositions. This makes the reading of *Perspectivae libri sex* a tedious and confusing experience.

Jean Étienne Montucla expressed the following opinion on Guidobaldo's *Perspectivae libri sex* in his *Histoire des mathématiques*.

> Moreover, the work by Guidobaldo suffers from the usual fault of its time; the matter it presents in a multitude of theorems could have been expressed far more neatly in fewer pages.[7]

While I agree with Montucla's observations on Guidobaldo's style, I disagree in calling it typical for his time. In fact, only five years after the publication of *Perspectivae libri sex*, the Dutch mathematician Simon Stevin presented the work's basic ideas in a few pages (Stevin 1605c). I rather relate the prolixity of

[7] "Au reste, l'ouvrage de Guido-Ubaldi a le défaut ordinaire de ceux de son temps; ce qu'on y trouve exposé en une multitude de propositions, pouvoit être dit avec plus de netteté en peu de pages" (Montucla 1758, vol.1, 636).

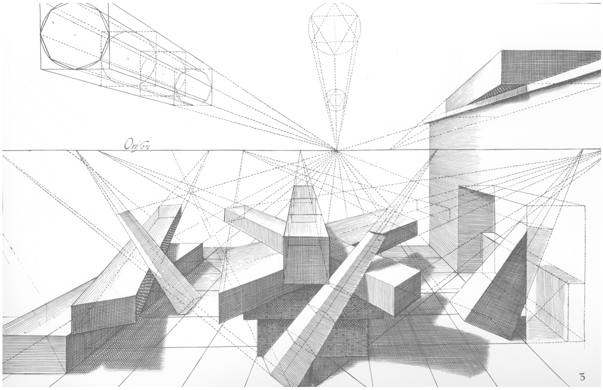

Figure 7.13: In this drawing Johan Vredeman de Vries placed the vanishing points of horizontal lines as well as of oblique lines on the horizon (De Vries 1604-1605, part II, Figure 3).

Guidobaldo's approach to his personal style and to the fact that the first formulations of concepts or ideas in mathematics are often more complicated than later presentations.

Considering the effort it takes to read *Perspectivae libri sex*, I suspect that the number of people who studied the work thoroughly is small; among those who have I would count Stevin, François Aguilon, and Samuel Marolois. Through their works Guidobaldo's ideas were spread to a wider audience (Stevin 1605c; Aguilon 1613 and Marolois 1614). Actually all the further developments of the theory of perspective have their roots in Guidobaldo's *Perspectivae libri sex*—a theme I return to in the conclusion in section 7.

Throughout this chapter I have concentrated on Guidobaldo's contributions to the theory of perspective, but when drawing the discussion to its end it seems appropriate to touch upon what his work meant to the practice of perspective. One of his successors, Claude François Milliet Dechales, surveyed the literature on perspective in 1690 and made the following remark on *Perspectivae libri sex*.

> The theory of this work is solid and geometrical, yet the method is somewhat on the difficult side. As a result, no one could learn perspective from this book alone; because it appears that it does not go sufficiently down into its practice. However, if one is moderately

> versed in perspective, one can gain much enlightenment from the book.[8]

Dechales was right, Guidobaldo's work is not helpful for non-mathematicians, but being written in Latin and mainly containing examples dealing with geometrical figures, it was not meant to be. Its direct influence on the practice of perspective is therefore minimal. However, it is my impression that Guidobaldo was instrumental in making many practitioners aware of the concept of a general vanishing point. Thus, in the literature published after the first decade of the seventeenth century, we do not find the misconception held by the Dutch artist Johan Vredeman de Vries repeated. He had somehow got the idea, as it can be seen in Figure 7.13, that all parallel lines—apart from those parallel to the picture plane—converge on the horizon.

7.7 Conclusion

In short, it can be concluded that an interest among Italian mathematicians in the latter half of the sixteenth century inspired Guidobaldo to take up perspective. He inherited the idea that not only orthogonals and diagonals have vanishing points, and through a gradual process, he came to realize the importance of the *main theorem* of perspective. His achievements were extremely important for the development of the mathematical theory of perspective and had some consequences for its practice. However, it was his results rather than his presentation that inspired his successors.

Let me add that Guidobaldo closed one era and opened another in the history of the mathematical theory of perspective in the sense that he provided the geometrical explanations behind perspective constructions and gave his successors inspiration to continue along his line. Thus, there is the following trajectory of inspiration (Andersen 2007, 717): Guidobaldo's insights travelled from Italy to the Northern Netherlands and were quickly adopted by Stevin and then picked up by Willem 'sGravesande. The latter did not add any decisive new results to the theory, but he appreciated and exploited its potential more strongly than his predecessors. Then, once more, the development crossed a border or, to be more precise, the North Sea to England. Taylor was inspired by 'sGravesande's mathematical understanding and added quite a bit of his own. In fact, Taylor provided perspective with a new mathematical life, among other things by introducing and applying the general concept of a vanishing line for a set of parallel planes cutting the picture plane. This line consists of the vanishing points of all the lines

[8] "Doctrina hujus operis solida est & geometrica, tamen methodus paulo difficilior; ita ut nullus in eo solo libro possit perspectivam addiscere, videtur enim non satis ad praxin descendisse, qui tamen in perspectiva mediocriter esset versatus posset ex eo multum lucis haurire," in (Dechales 1690, 68).

in the parallel planes—the horizon being a noticeable example of a vanishing line, namely of horizontal planes. Guidobaldo did not single out the concept of a vanishing line, but it occurs implicitly in his work (Andersen 2007, 249).

The story about the development of the mathematical theory of perspective ends with Lambert because he left no important questions about how to project three-dimensional figures upon a plane surface unanswered. Lambert's approach to perspective was to perform all constructions directly in the picture plane; in doing so he fully elaborated one of Guidobaldo's contributions to perspective.

Acknowledgement

I am extremely thankful to Henk Bos for having commented upon this paper and not least for having drawn the Figures 7.1–5, 7.11 and 7.12.

References

Aguilon, F. (1613). De scenographice. In: *Opticorum libri sex*. Antwerpen: Officina Plantiniana, 637–681.

Alberti, L. B. (1972). *On painting and on sculpture. The latin texts of De pictura and De statua*. Ed. by C. Grayson. London: Phaidon.

Andersen, K. (2007). *The Geometry of an Art: the History of the Mathematical Theory of Perspective from Alberti to Monge*. New York: Springer.

Andersen, K. and E. Gamba (2008). Monte, Guidobaldo, Marchese Del. In: *New Dictionary of Scientific Biography*. Ed. by N. Koertge. Vol. 5. Detroit: Scribners' Sons, 174–178.

Benedetti, G. B. (1585). *Diversarum speculationum mathematicarum, et physicarum liber: quarum seriem sequens pagina indicabit*. Torino: Bevilaqua.

Commandino, F. (1558). *Federici Commandini urbinatis in Planisphaerium Ptolemaei commentarius*. Venezia: Aldo Manuzio.

Dechales, C. F. M. (1690). *Cursus seu mundus mathematicus*. Ed. by A. Varcin. 4. Lyon: Posuel Rigaud.

De Vries, Johann V. (1604-1605). *Perspective: das ist die weit beruehmte Khunst*. Leiden: Hondius.

Galilei, G. (1929-1939). *Le opere di Galileo Galilei*. Ed. by A. Favaro. Firenze: Barbèra.

Marchi, P. (1998). *L'invenzione del punto di fuga nell'opera prospettica di Guidobaldo dal Monte*. MA thesis. Pisa: Università degli studi di Pisa.

Marolois, S. (1614). *La perspective contenant la theorie et la pratique*. Den Haag: H. Hondius.

Monte, Guidobaldo del (1600). *Perspectivae libri sex*. Pesaro: Girolamo Concordia.

Montucla, J. E. (1758). *Histoire des mathématiques*. Vol. 2. Paris: Jombert.

Piero, della Francesca (1942). *De prospectiva pingendi*. Ed. by G. Nicco Fasola. Vol. 1. Firenze: Sansoni.

Rose, P. L. (1974). Monte, Guidobaldo, Marchese Del. In: *Dictionary of Scientific Biography*. Ed. by C. C. Gillispie. Vol. 9. New York: Scribners' Sons, 487–489.

Sinisgalli, R. (1978). *Per la storia della prospettiva (1405-1605). Il contributo di Simon Stevin allo sviluppo scientifico della prospettiva artificiale ed i suoi precedenti storici*. Roma: L'Erma di Bretschneider.

Stevin, S. (1605a). De optica. In: *Hypomnemata mathematica a Simone Stevino. Tomus tertius*. Leiden: Ex Officina Ioannis Patii.

— (1605b). Des perspectives. In: *Mémoires mathématiques par Simon Stevin. Livre trois*. Leiden: Paedts Iacobsz.

— (1605c). Van de verschaeuwing. Eerste bouck der deursichtighe. In: *Wisconstighe Ghedachtnissen*. Leiden.

Taylor, B. (1715). *Linear Perspective: or, a New Method of Representing justly all Manner of Object*. London.

— (1719). *New Principles of Linear Perspective: or the Art of Designing on a Plane the Representations of all Sorts of Objects, in a more General and Simple Method than has been done before*. London.

Vignola, J. Barozzi da (1583). *Le due regole della prospettiva pratica di M. Iacomo Barozzi da Vignola, con i comentarij del R.P.M. Egnatio Danti dell'ordine de predicatori, matematico dello studio di Bologna*. Roma: Francesco Zannetti.